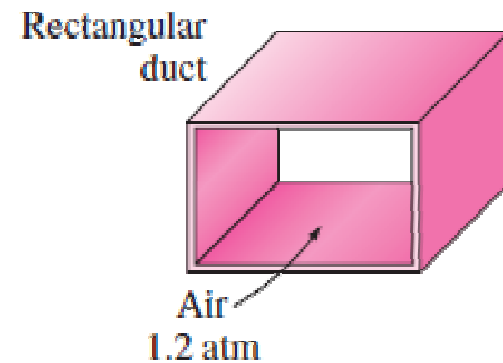
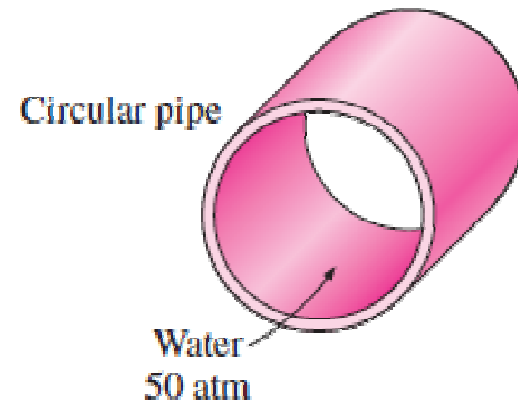


Turbulent Flow in Pipes

Dr. Sanghamitra Kundu

- In general, flow sections of circular cross section are referred to as **pipes** (especially when the fluid is a liquid)
- flow sections of noncircular cross section are referred to as **ducts** (especially when the fluid is a gas).
- Smaller diameter pipes are usually referred to as **tubes**.



Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any distortion, but noncircular pipes cannot.

Theoretical Discussion

- ▶ Fluid flow in pipes is of considerable importance in process.
 - ▶ Animals and Plants circulation systems.
 - ▶ In our homes.
 - ▶ City water.
 - ▶ Irrigation system.
 - ▶ Sewer water system



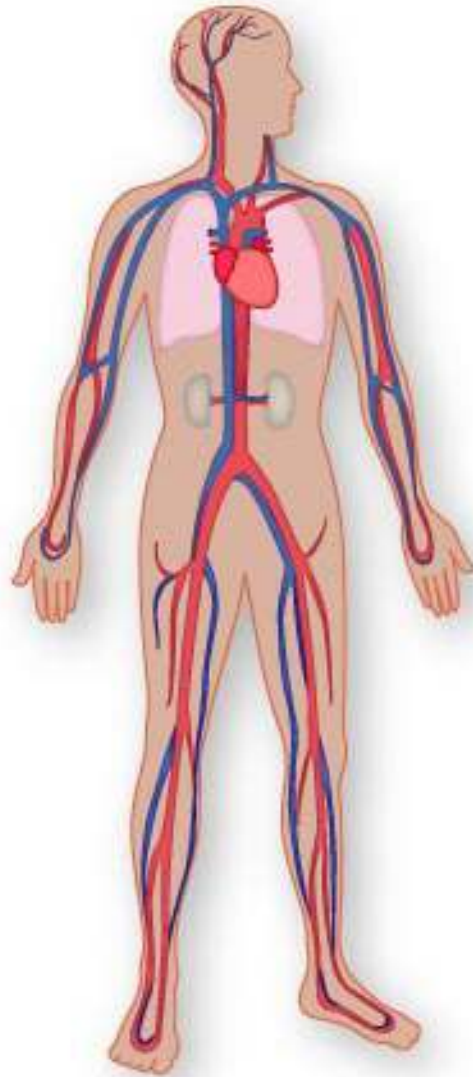
Distribution of liquids



Oil and natural gas pipelines



Blood flow through arteries and veins



Laminar vs Turbulent

- ▶ **Turbulence is of importance in the mixing of fluids.**
 - ▶ Smoke from a stack would continue for miles as a ribbon of pollutant without rapid dispersion within the surrounding air if the flow were laminar rather than turbulent. Under certain atmospheric conditions this is observed to occur. Although there is mixing on a molecular scale (laminar flow), it is several orders of magnitude slower and less effective than the mixing on a macroscopic scale (turbulent flow).
 - ▶ It is considerably easier to mix cream into a cup of coffee (turbulent flow) than to thoroughly mix two colors of a viscous paint (laminar flow).
- ▶ **In other situations laminar (rather than turbulent) flow is desirable.**
 - ▶ The pressure drop in pipes (hence, the power requirements for pumping) can be considerably lower if the flow is laminar rather than turbulent.
 - ▶ Fortunately, the blood flow through a person's arteries is normally laminar, except in the largest arteries with high blood flowrates.
 - ▶ The aerodynamic drag on an airplane wing can be considerably smaller with laminar flow past it than with turbulent flow.



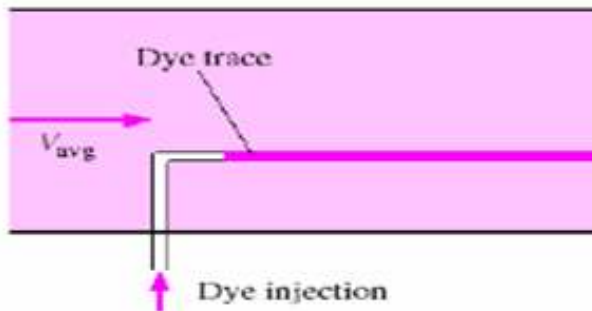
Laminar and Turbulent Flows

Laminar Flow

Can be steady or unsteady.

(Steady means that the flow field at any instant in time is the same as at any other instant in time.)

Has regular, *predictable* behavior



Analytical solutions are possible

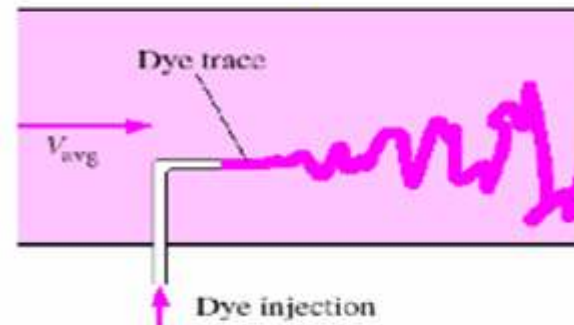
Occurs at *low* Reynolds numbers.

Turbulent Flow

Is always *unsteady*.

Why? There are always random, swirling motions (vortices or eddies) in a turbulent flow.

Has irregular or *chaotic* behavior (cannot predict exactly – there is some randomness associated with any turbulent flow.)



No analytical solutions exist! (It is too complicated, again because of the 3-D, unsteady, chaotic swirling eddies.)

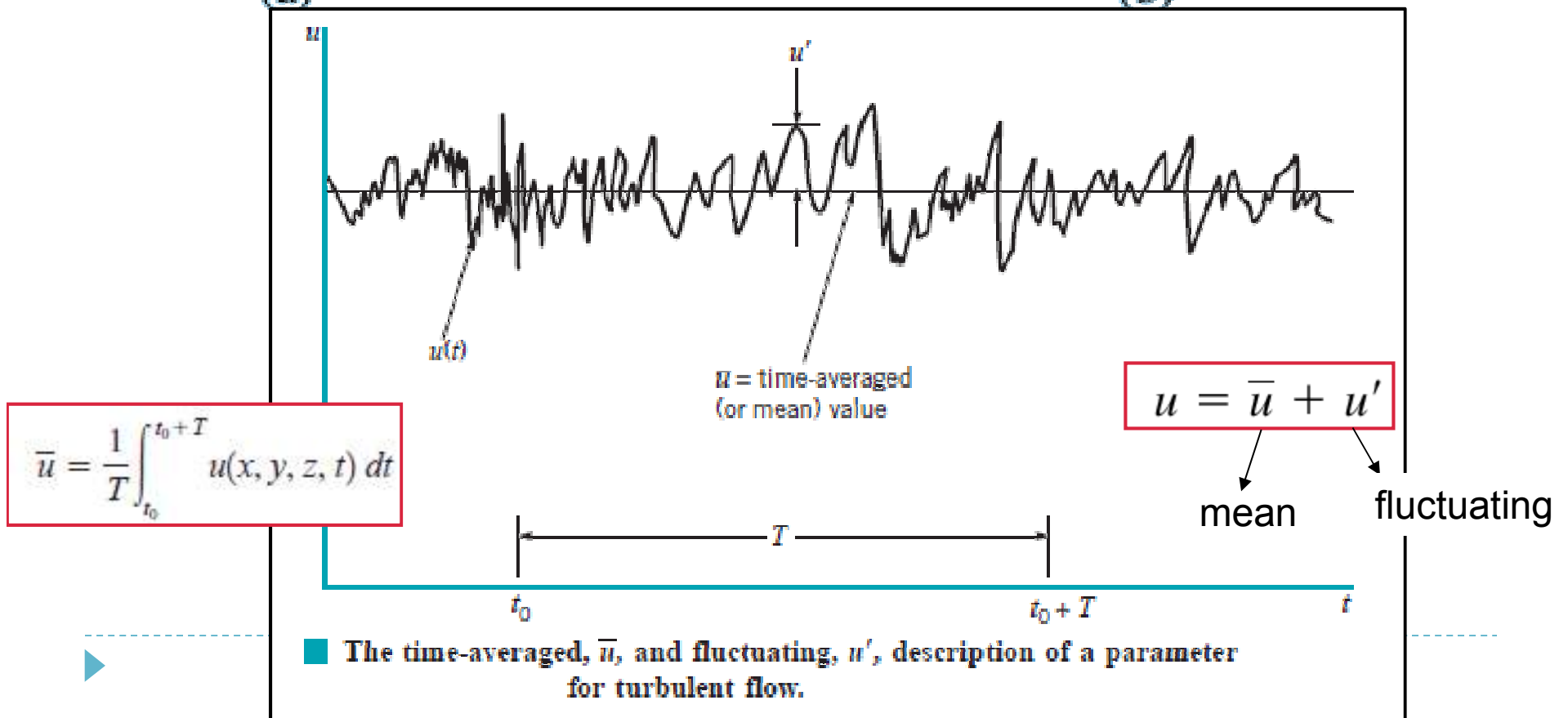
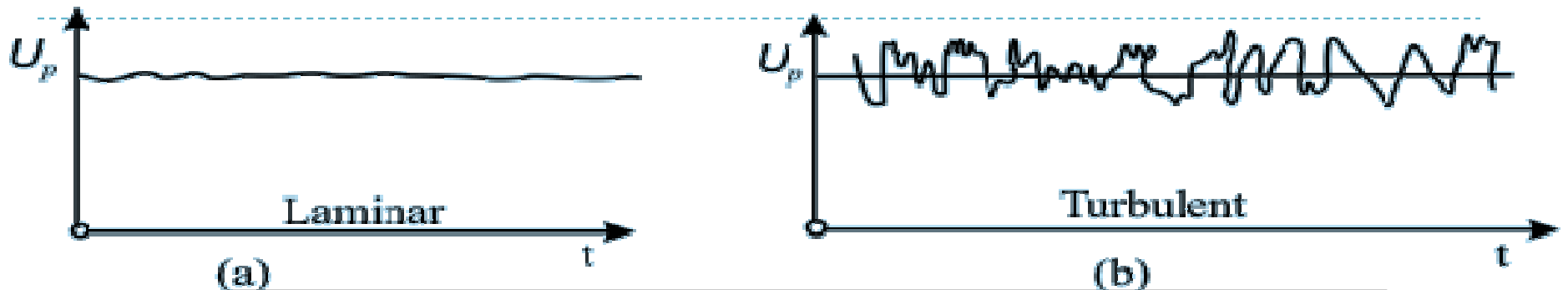
Occurs at *high* Reynolds numbers.

Introduction

- ▶ A turbulent flow is one that has:
 - ▶ Most commonly encountered pipe flow
 - ▶ Without turbulence it would be virtually impossible to carry out life as we now know it.
 - ▶ Mixing is one positive application of turbulence, as discussed above, but there are other situations where turbulent flow is desirable. To transfer the required heat between a solid and an adjacent fluid (such as in the cooling coils of an air conditioner or a boiler of a power plant) would require an enormously large heat exchanger if the flow were laminar. Similarly, the required mass transfer of a liquid state to a vapor state (such as is needed in the evaporated cooling system associated with sweating) would require very large surfaces if the fluid flowing past the surface were Laminar
 - ▶ Reynolds Number greater than 4000
 - ▶ Uniform velocity distribution when compared to laminar flow
 - ▶ Eddy currents due to their haphazard movements causing complete mixing of the fluid.
 - ▶ Turbulent Shear: it is the additional shear (frictional) resistance caused due to velocity fluctuations influencing the mean motion of fluid.



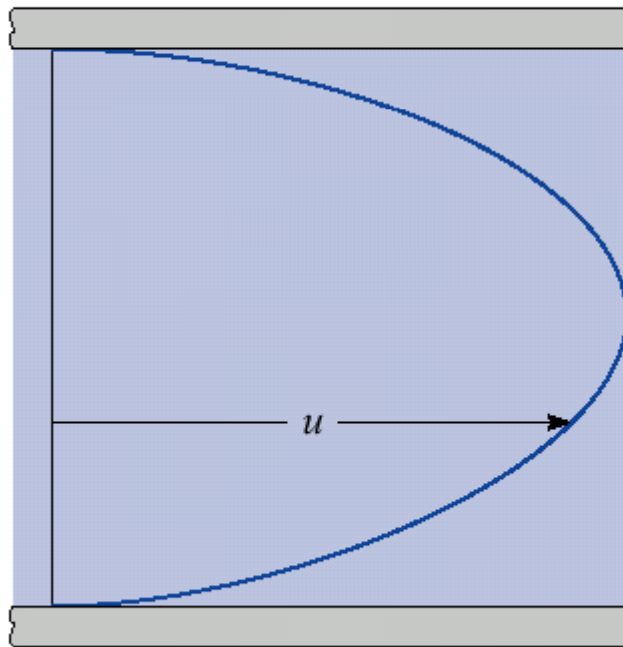
a. Velocity and pressure fluctuate with time



Laminar vs Turbulent Flow

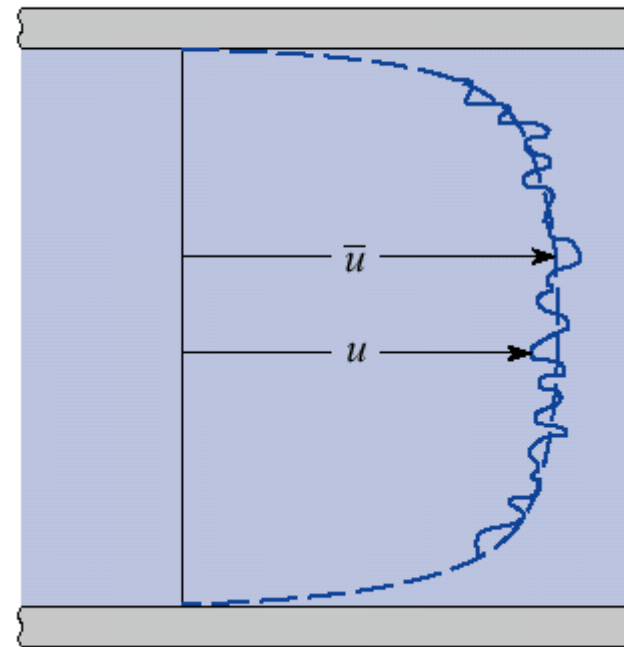
b. Uniform velocity distribution in turbulent flow as compared to laminar flow.

▶ Laminar



(a)

▶ Turbulent

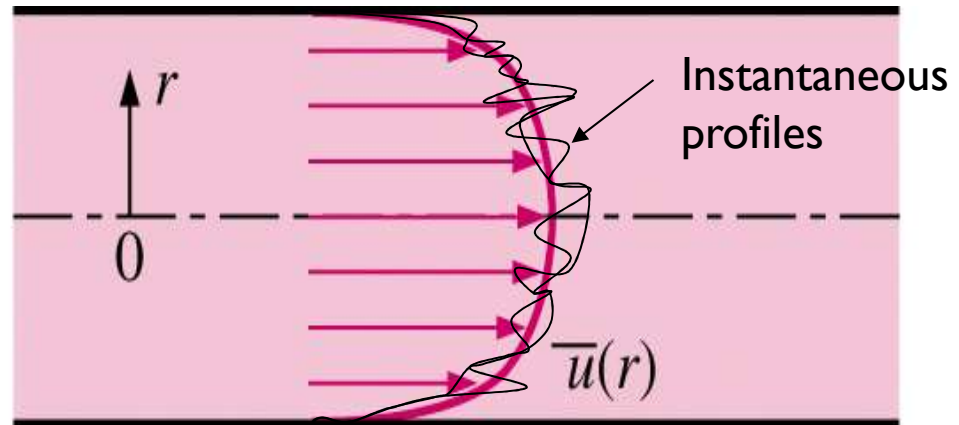


(b)

Fully Developed Pipe Flow

Turbulent

- ▶ Cannot be solved exactly (too complex)
- ▶ Flow is unsteady (3D swirling eddies)
- ▶ Mean velocity profile is fuller (shape more like a top-hat profile, with very sharp slope at the wall)
- ▶ Pipe roughness is very important
- ▶ No analytical solution



Shear Stresses in Turbulent Flow.

- ▶ Velocity fluctuations cause interchange of fluid masses between the neighboring layers, which is accompanied by a transfer of momentum
- ▶ The momentum transfer is because each fluid layer possesses a different velocity and a large number of lumps of fluid particles move from one fluid layer into adjacent layers above and below.
- ▶ This change of momentum is equivalent to the force in particular direction.
- ▶ Hence such momentum transport due to fluctuations result in developing additional shear stress of high magnitude between adjacent layers.



Semi-empirical Theories to determine Turbulent Shear

J. Boussinesq's Equation
(1877)

$$\tau = (\mu + \eta) \frac{d\bar{u}}{dy}$$

where;

τ turbulent shear stress

μ absolute viscosity

η turbulent mixing coefficient (eddy viscosity)

\bar{u} average velocity at a distance y from boundary.



-
- ▶ The value of η may vary from zero (if laminar flow) to several thousand times of μ .
 - ▶ The value of η depends on the momentum carried by the migrating particles and thus on the density of the flowing fluid and the characteristics of flow.
 - ▶ Further the kinematic eddy viscosity $\varepsilon = \frac{\eta}{\rho}$ may be considered to be independent of the properties of the fluid but depends on characteristics of the flow.
 - ▶ Since, the values of η and ε cannot be predicted, the Boussinesq's hypothesis is, however, of limited use.

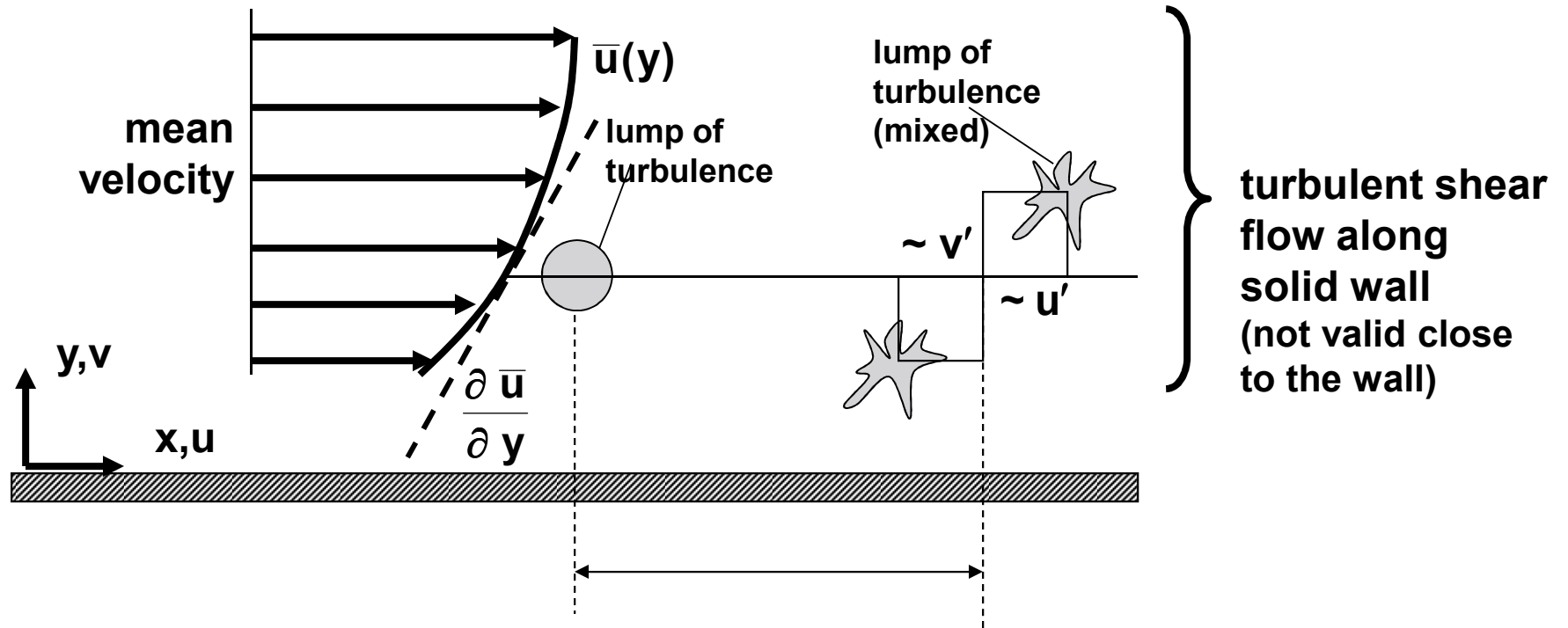


2. Prandtl's Mixing Length

- ▶ Mixing length is that distance in the transverse direction which must be covered by a lump of particles travelling with its original mean velocity in order to make the difference between its velocity and the velocity of the new layer equal to the mean transverse fluctuation in turbulent flow.



Prandtl's Mixing Length Theory



$$v' \sim +l_1 \cdot \frac{\partial \bar{u}}{\partial y} \Rightarrow u' \sim \pm l_2 \cdot \frac{\partial \bar{u}}{\partial y}$$

$$l_1 = l_2 = l$$

mixing length l defined as that distance, which is needed for the lump of turbulence to be completely mixed with the surrounding fluid

Turbulence is even in all directions

► (homogeneous)

- ▶ u' , v' are the velocity fluctuation in x-direction and y direction respectively

$$u' = l \frac{\partial \bar{u}}{\partial y}$$

$$v' = l \frac{\partial \bar{u}}{\partial y}$$

- ▶ Now, $u' \times v'$ can be written as :

$$\overline{u'v'} = l^2 \left(\frac{\partial \bar{u}}{\partial y} \right)^2$$

- ▶ The equation for turbulent shear becomes

$$\bar{\tau} = \rho l^2 \left| \frac{\partial \bar{u}}{\partial y} \right|^2$$

- ▶ Thus, the equation for total shear stress at any point is the sum of viscous shear stress and turbulent shear stress and expressed as follows:

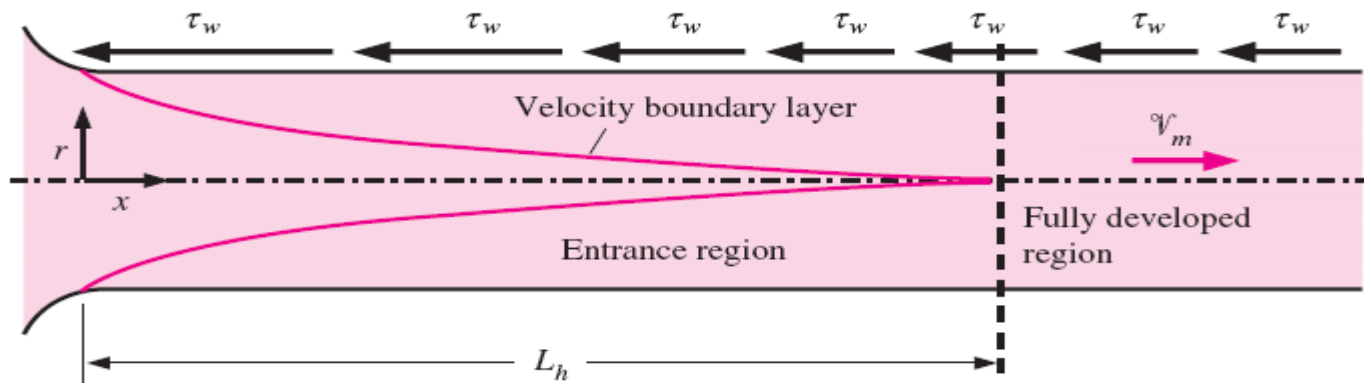
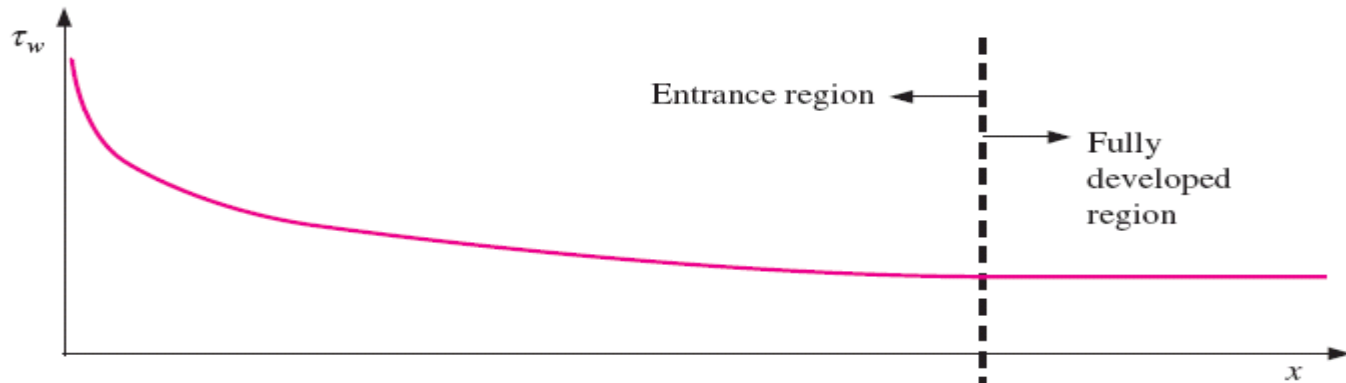
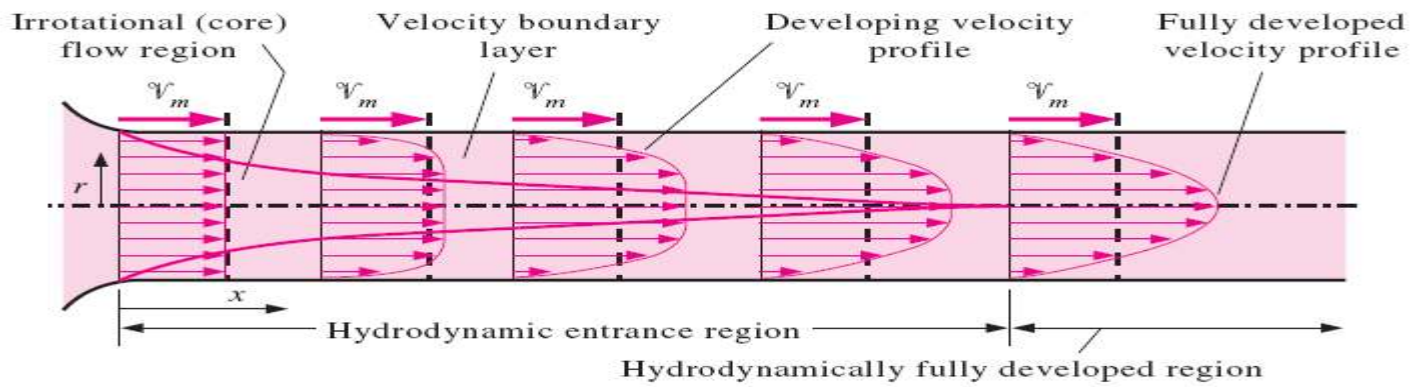
$$\bar{\tau} = \mu \frac{d\bar{u}}{dy} + \rho l^2 \left(\frac{d\bar{u}}{dy} \right)^2$$



Formation of Boundary Layer

- ▶ When the fluid flow through pipe, close to the pipe wall the fluid is retarded, thus resulting in formation of boundary layer.
- ▶ The boundary layer may attain a maximum thickness equal to radius of pipe.
- ▶ This is because at entrance section of pipe, the boundary layer gradually increases and at a certain section in the downstream when it attains thickness equal to radius it cannot expand more.
- ▶ With increase in the thickness of the boundary layer, more fluid will be retarded in the downstream direction, and hence in order to maintain a constant discharge, the velocity in central core will increase





-
- ▶ The hydrodynamic entry length is usually taken to be the distance from the pipe entrance where the friction factor reaches within about 2 percent of the fully developed value. In *laminar flow*, the hydrodynamic entry length is given approximately as

$$x/D = 0.07Re$$

where,

x= length from entrance upto section AA

D= Diameter of pipe

Re= Reynolds number

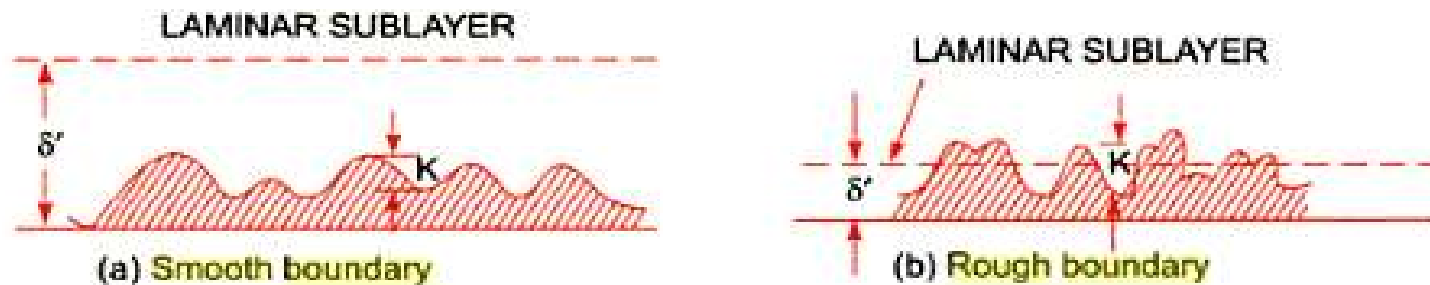


- ▶ If the flow in the pipe is turbulent, for small distance from entrance section, laminar will change to turbulent boundary layer before the thickness of the boundary layer becomes equal to radius of pipe.
- ▶ However, in some cases, if the pipe is rough and the intensity of incoming turbulence is high, from the entrance section itself turbulent boundary layer will be formed.
- ▶ The thickness of the boundary layer will increase in downward direction upto certain section AA.
- ▶ From section AA onwards a fully developed turbulent flow will exist in the pipe.
- ▶ Since the a turbulent flow has a logarithmic velocity distribution, it is much more uniform and hence length of pipe required for establishment of fully developed turbulent flow is less. Rouse has indicated as follows: $x/D = 50$.



Hydrodynamically Smooth and Rough Boundaries

- ▶ A boundary with irregularities of large average height k , on its surface is considered to be a rough boundary and one with small values of k is considered as smooth.



- ▶ With the increase in Reynolds number, the thickness of the laminar sub-layer decreases, and it can even become much smaller than the average height k , of surface irregularities. The irregularities will then project through the laminar sub-layer and laminar sub-layer is completely destroyed. The eddies will thus come in contact with the surface irregularities and large amount of energy loss will take place and thus the boundary act as a rough boundary.

From Nikuradse's experiment

- ▶ Hydrodynamically smooth pipe


$$\frac{k}{\delta'} < 0.25$$

- ▶ Transition region in a pipe

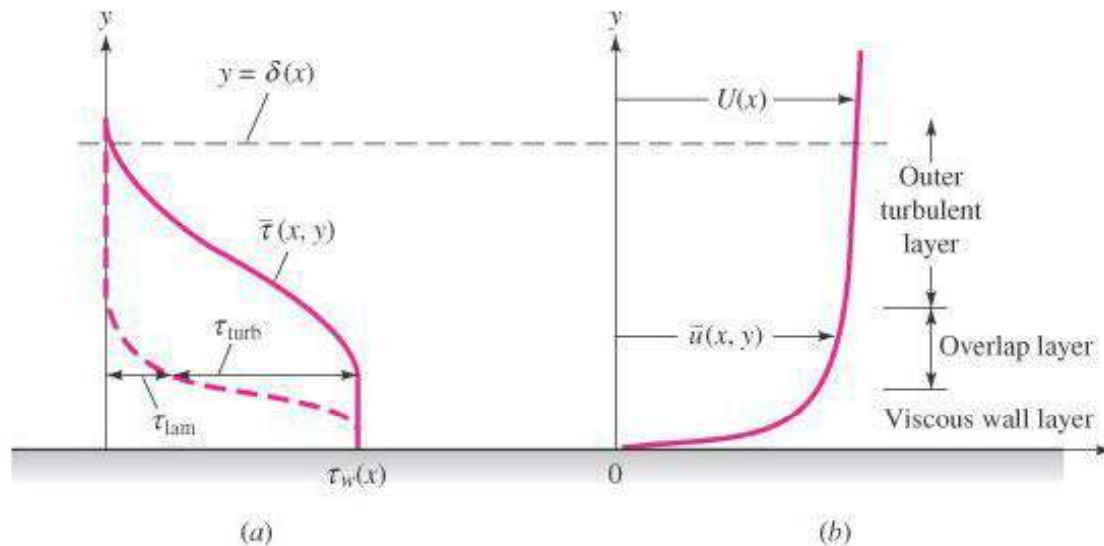
$$0.25 < \frac{k}{\delta'} < 6.0$$

- ▶ Hydrodynamically Rough Pipe

$$\frac{k}{\delta'} > 6.0$$



Typical velocity and shear distributions in turbulent flow near a wall: (a) shear; (b) velocity.



Wall layer: Viscous shear dominates.

Outer layer: Turbulent shear dominates.

Overlap layer: Both types of shear are important.

- ▶ Laminar shear is dominant near the wall (the *wall layer*), and turbulent shear dominates in the *outer layer*. There is an intermediate region, called the *overlap layer*, where both laminar and turbulent shear are important.
- ▶ In the outer layer τ_{turb} is two or three orders of magnitude greater than τ_{lam} , and vice versa in the wall layer. These experimental facts enable us to use a crude but very effective model for the velocity distribution $\bar{u}(y)$ across a turbulent wall layer.

Velocity Distribution for Turbulent Flow

- ▶ For turbulent flow in circular pipes, Prandtl assumed the mixing length l to be linear function of the distance y from the pipe wall i.e $l \sim y$ so, $l = \kappa y$ where κ is constant of called 'Karman universal constant.'

- ▶ J. Nikuradse experimental results has shown that in immediate neighborhood of pipe boundary the expression for mixing length is

$$l = 0.4y - 0.44(y^2 / R)$$

- ▶ For small values of y , close to pipe boundary, above equation reduces to $l = 0.4y$



- ▶ Now substituting for mixing length the turbulent shear stress may be expressed as:

$$\bar{\tau} = \rho l^2 \left(\frac{\partial \bar{u}}{\partial y} \right)^2 \quad \longrightarrow \quad \bar{\tau} = \rho \kappa^2 y^2 \left(\frac{\partial \bar{u}}{\partial y} \right)^2$$

- ▶ Analogously considering shear stress to vary linearly with radius

$$\tau = \tau_0 \left(\frac{r}{R} \right) = \tau_0 \left(\frac{R-y}{R} \right) = \tau_0 \left(1 - \frac{y}{R} \right)$$

- ▶ For very small values of y , $\tau \sim \tau_0$

$$\frac{\tau_0}{\rho} = \kappa^2 y^2 \left(\frac{\partial u}{\partial y} \right)^2 \quad \Rightarrow \quad \frac{\partial u}{\partial y} = \frac{1}{\kappa y} \left(\frac{\tau_0}{\rho} \right)^{1/2} = \frac{U^*}{\kappa y}$$



-
- ▶ Since U^* is shear velocity or friction velocity , it will be constant for a given case of turbulent flow. Hence the above equation may be integrated to yield following relationship

$$u = \frac{U^*}{\kappa} \log_e y + C$$

The turbulent flow velocity thus varies directly with log of the distance from the boundary or the velocity distribution is logarithmic in nature.

- ▶ C constant of integration may be determined from boundary condition : $u = u_{\max}$ at $y=R$

$$C = u_{\max} - \frac{U^*}{\kappa} \log_e R$$



$$u = u_{\max} + 2.5U^* \log_e \left(\frac{y}{R} \right)$$

Applicable to both hydrodynamically smooth and rough pipes as no assumption has been made in this regard



► Non-dimensional form

$$\frac{u_{\max} - u}{U^*} = 2.5 \log_e \left(\frac{R}{y} \right) \quad \longrightarrow \quad \frac{u_{\max} - u}{U^*} = 5.75 \log_{10} \left(\frac{R}{y} \right)$$

Velocity Defect

$$\frac{u_{\max} - u}{U^*} \sim f(R/y) \quad \longrightarrow \quad \text{Independent of the nature of boundary}$$



Velocity distribution for turbulent flow in Hydrodynamically smooth and rough pipes

$$u = \frac{U^*}{\kappa} \log_e \left(\frac{y}{y'} \right)$$

$$U^{*2} = \frac{\nu u}{y}$$



$$\frac{u}{U^*} = \frac{U^* y}{\nu}$$

Valid for $y = 0$ to $y = \delta'$

11.6 for $y = \delta'$

0.108 for $y = y'$

$$\frac{u}{U^*} = 5.75 \log_{10} \left(\frac{U^* y}{\nu} \right) + 5.5$$

For smooth pipes

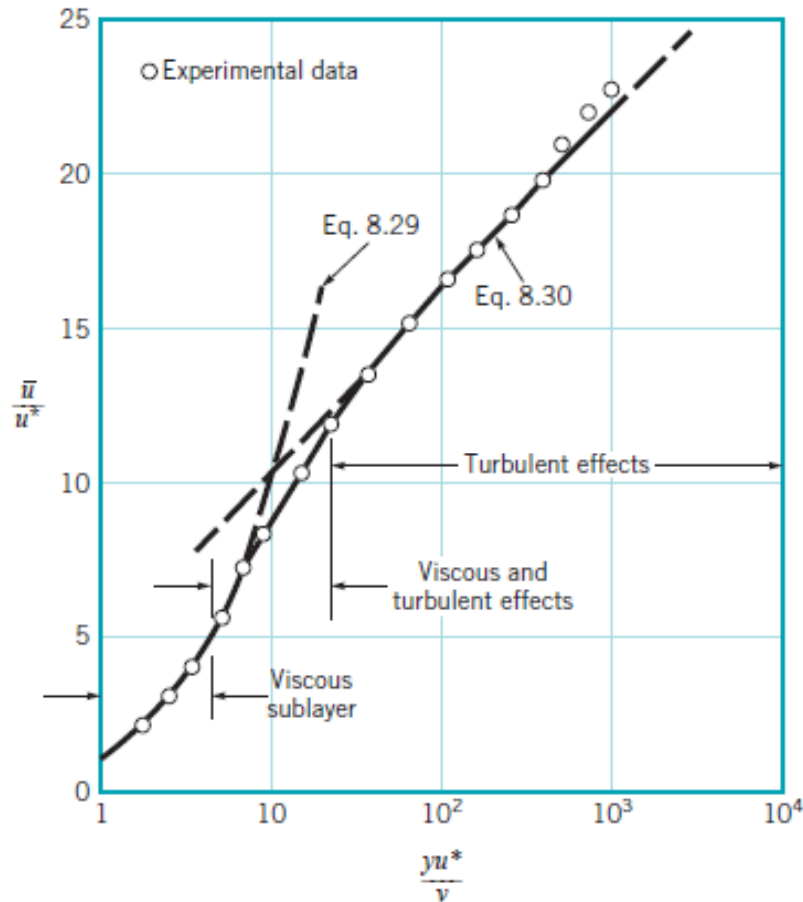
$$\frac{u}{U^*} = 5.75 \log_{10} \left(\frac{y}{k} \right) + 8.5$$

For rough pipes

$$y' = k/30$$



Velocity distribution for turbulent flow in Hydrodynamically smooth and rough pipes



$$\frac{\bar{u}}{u^*} = \frac{yu^*}{\nu} \quad (8.29)$$

$$\frac{\bar{u}}{u^*} = 2.5 \ln\left(\frac{yu^*}{\nu}\right) + 5.0 \quad (8.30)$$

Experimental verification of the inner, outer, and overlap layer laws relating velocity profiles in turbulent wall flow.

-
- ▶ Hydrodynamically smooth Plate

$$\frac{U^* k}{\nu} < 5$$

- ▶ Plate in Transition region

$$5 < \frac{U^* k}{\nu} < 70$$

- ▶ Hydrodynamically Rough

$$\frac{U^* k}{\nu} > 70$$

Where k is equivalent sand grains roughness defined as that value of the roughness which would offer the same resistance to the flow past the plate as that of due to the actual roughness on the surface of the plate.



Velocity Distribution for turbulent flow in terms of Mean Velocity (V)

- ▶ Velocity Distribution in a hydrodynamically smooth pipe

$$\frac{U}{U^*} = 5.75 \log_{10} \frac{U^* R}{\nu} + 1.75$$

- ▶ Velocity Distribution in a hydrodynamically Rough Pipe

$$\frac{U}{U^*} = 5.75 \log_{10} \left(\frac{R}{k} \right) + 4.75$$



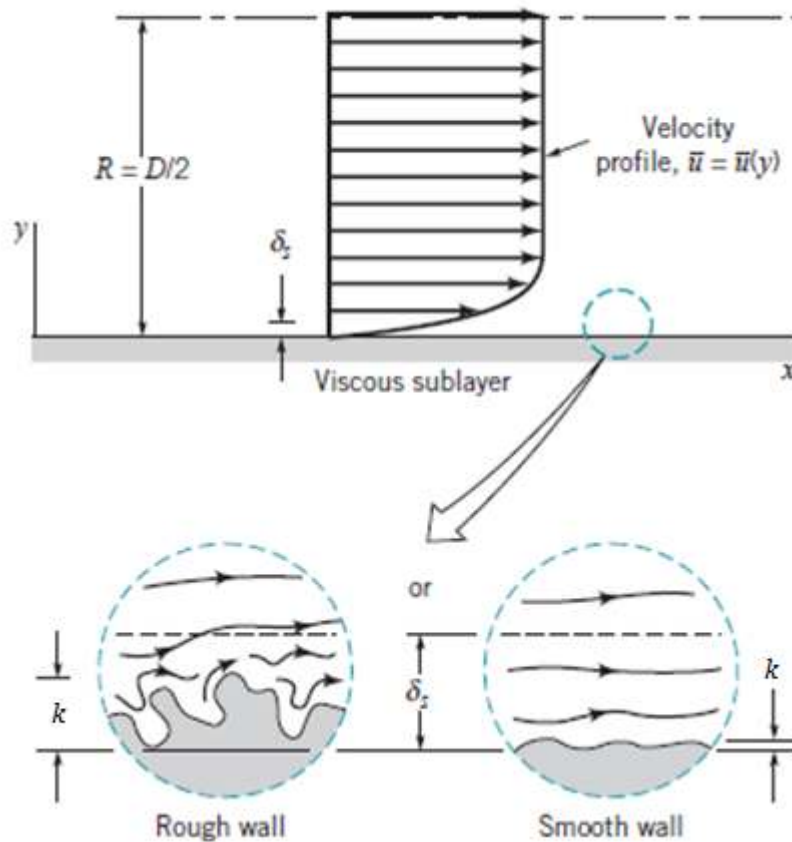
Resistance to Flow of Fluids

- ▶ When a fluid flows through a pipe frictional resistance is offered to the motion of the fluid
 - ▶ Dissipation of certain amount of energy
 - ▶ Reduction of pressure intensity in direction of flow
- ▶ Although the pressure drop for laminar pipe flow is found to be independent of the roughness of the pipe, it is necessary to include this parameter when considering turbulent flow.
- ▶ If a typical wall roughness element protrudes sufficiently far into (or even through) this layer, the structure and properties of the viscous sublayer (alongwith Δp and τ) will be different than if the wall were smooth. Thus, for turbulent flow the pressure drop is expected to be a function of the wall roughness. For laminar flow there is no thin viscous layer—viscous effects are important across the entire pipe.



Resistance to Flow of Fluids

- ▶ The loss of head due to friction may be correctly predicted if friction factor f can be evaluated accurately.



Flow in the viscous sublayer near rough and smooth walls.

$$f = \phi \left(\frac{UD}{\nu}, \frac{k}{D} \right)$$

Reynold's Number

Relative roughness

Resistance to Flow of Fluids

- ▶ Variation of friction factor f for laminar flow

$$f = \frac{64}{R_e}$$

- ▶ Friction factor varies inversely with Reynolds number and independent of relative roughness.
- ▶ This is valid upto $R_e = 2000$ which is also limit for laminar flow in circular pipes.



Resistance to Flow of Fluids

- ▶ Variation of Friction Factor for Turbulent Flow
 - ▶ In transition zone i.e $2000 < Re < 4000$, there does not exist any specific relation between Re and f .
 - ▶ If $Re > 4000$, the friction factor depends on Re or relative roughness depending on the boundary conditions.
 - ▶ Turbulent flow in smooth pipes
 - ▶ Turbulent flow in rough pipes
- ▶ However, the functional dependence of the friction factor on the Reynolds number and the relative roughness, is a rather complex one that cannot, as yet, be obtained from a theoretical analysis.
- ▶ Much of this information is a result of exhaustive set experiments conducted by J. Nikuradse in 1933 and amplified by many others since then.



Resistance to Flow of Fluids

$$h_{L \text{ major}} = f \frac{\ell}{D} \frac{V^2}{2g}$$

- ▶ One difficulty lies in the determination of the roughness of the pipe. Nikuradse used artificially roughened pipes produced by gluing sand grains of known size onto pipe walls to produce pipes with sandpaper-type surfaces.
 - ▶ The pressure drop needed to produce a desired flowrate was measured and the data were converted into the friction factor for the corresponding Reynolds number and relative roughness.
 - ▶ The tests were repeated numerous times for a wide range of Re and k/D to determine the $f = \varphi(Re, k/D)$ dependence.
-



Variation of Friction Factor for Turbulent Flow in Smooth Pipes

► Blasius (1911)

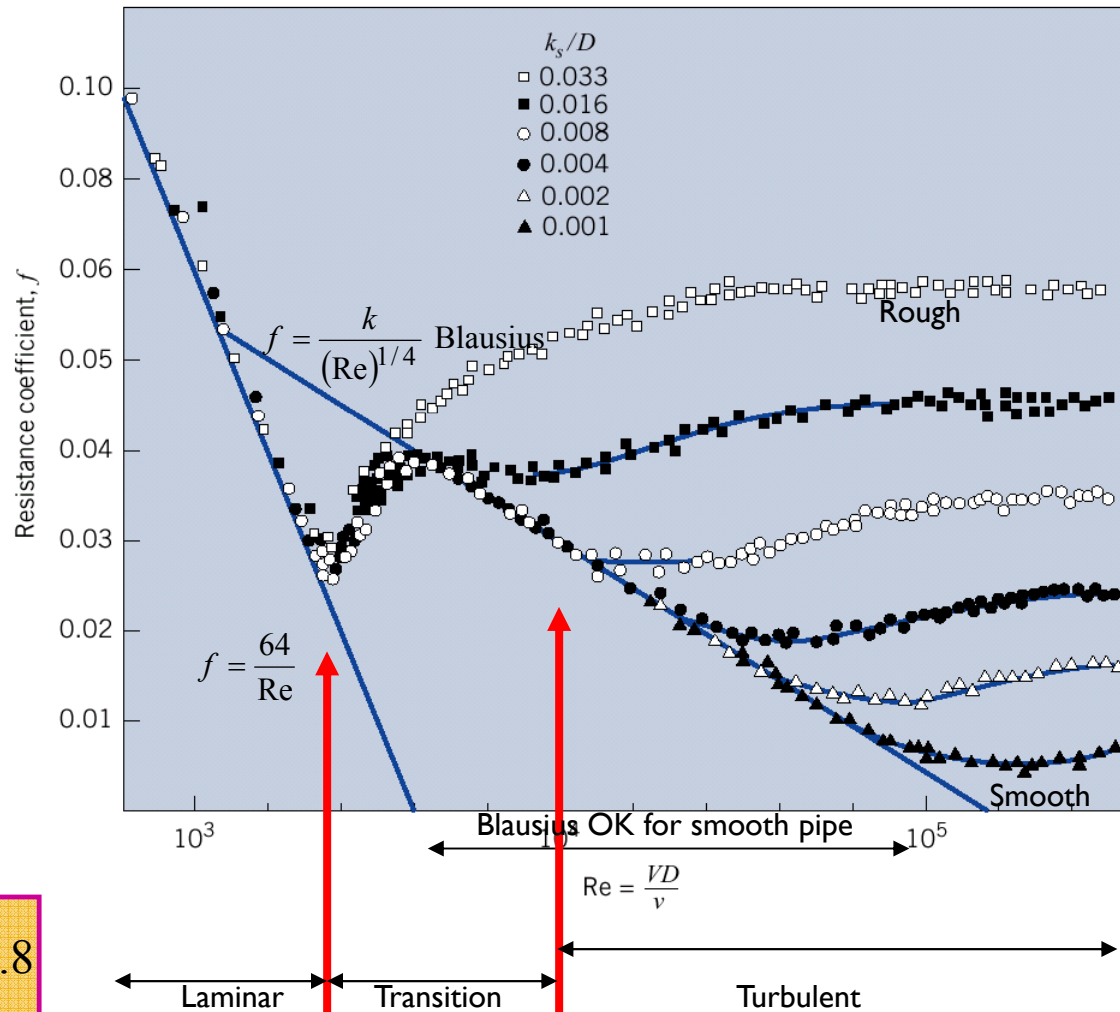
$$f = \frac{0.316}{(Re)^{1/4}}$$

($4000 < Re < 10^5$)

- By simply computing the mean velocity from the logarithmic law correlation, Prandtl derived an equation in 1935 and then adjusted the constants slightly to fit friction data better

$$\frac{1}{\sqrt{f}} = 2.0 \log(Re \sqrt{f}) - 0.8$$

($5 \times 10^4 < Re < 4 \times 10^7$)



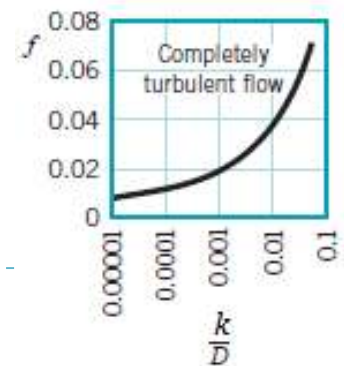
- ▶ Alternatively used to determine f directly (Nikuradse)

$$f = 0.0032 + \frac{0.221}{(\text{Re})^{0.237}}$$

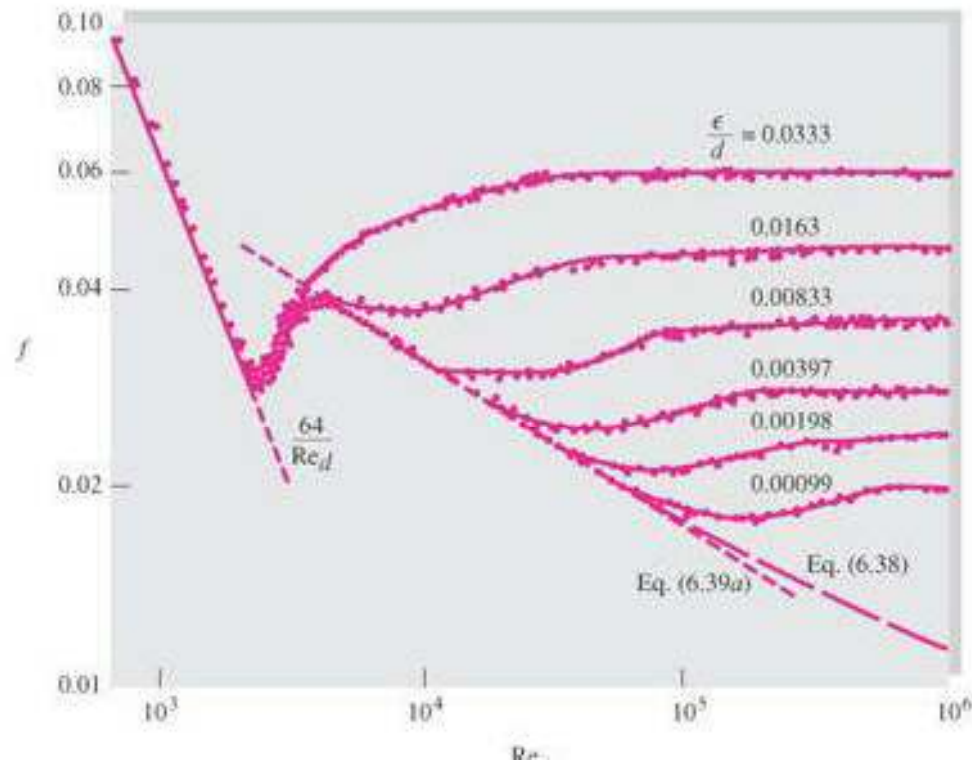
Variation of Friction Factor for Turbulent Flow in **Rough Pipes**

- ▶ For turbulent flows with very large Reynolds numbers, is independent of the Reynolds number. For such flows, commonly termed *completely turbulent flow* (or *wholly turbulent flow*), the laminar sublayer is so thin (its thickness decreases with increasing Re) that the surface roughness completely dominates the character of the flow near the wall

$$\frac{1}{\sqrt{f}} = 2.0 \log \left(\frac{R}{k} \right) + 1.74$$



Variation of Friction Factor for Turbulent Flow in Rough Pipes



Greater the value of k/D , lesser is the Reynolds number at which the pipe which originally behaves as a smooth pipe begins to follow the pattern of a rough pipe

Criteria for Hydrodynamically Smooth and Rough Boundaries

$$\frac{k}{\delta'} = \frac{\text{Re} \sqrt{f}}{65.6(R/k)}$$

- ▶ Hydrodynamically smooth

$$\frac{\text{Re} \sqrt{f}}{(R/k)} < 17$$

- ▶ Hydrodynamically Rough

$$\frac{\text{Re} \sqrt{f}}{(R/k)} > 400$$



Variation of Friction Factor for Turbulent Flow in **Rough Pipes**

- ▶ There is no Reynolds number effect; hence the head loss varies exactly as the square of the velocity in this case. Some numerical values of friction factor may be listed:

k/d	0.00001	0.0001	0.001	0.01	0.05
f	0.00806	0.0120	0.0196	0.0379	0.0716

- ▶ The friction factor increases by 9 times as the roughness increases by a factor of 5000. In the transitional roughness region, sand grains behave somewhat differently from commercially rough pipes, so the [graph](#) (shown in the previous slide) has now been replaced by the Moody chart.
-



Moody's Chart

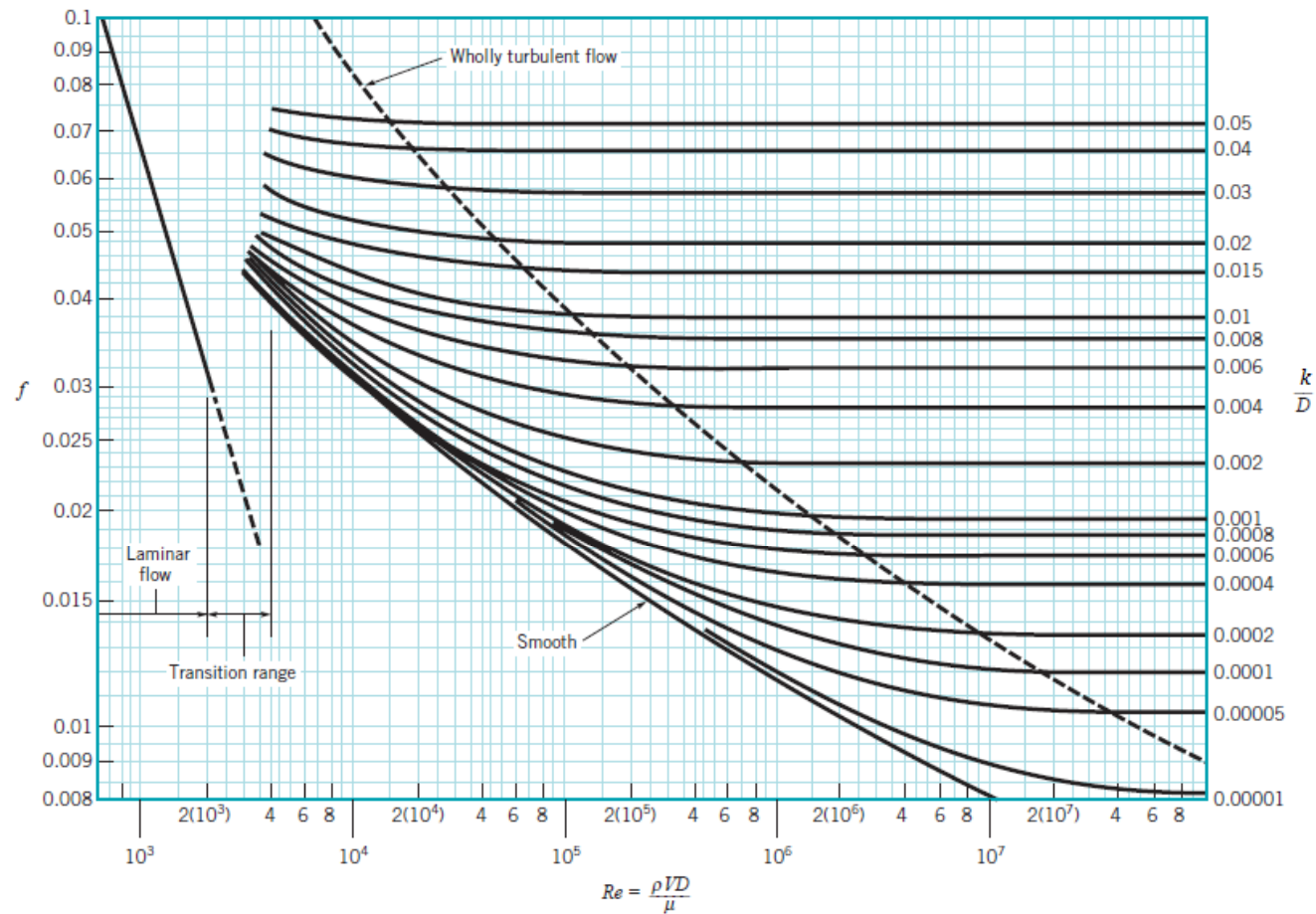
- ▶ In 1939 to cover the transitionally rough range, Colebrook combined the smooth wall and fully rough relations of Prandtl and Nikuradse into a clever interpolation formula

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{k/D}{3.7} + \frac{2.51}{\text{Re} \sqrt{f}} \right)$$

- ▶ Also called ***Colebrook formula***
- ▶ This is the accepted design formula for turbulent friction. It was plotted in 1944 by Moody into what is now called the Moody chart for pipe friction. The Moody chart is probably the most famous and useful figure in fluid mechanics. It is accurate to ± 15 percent for design calculations over the full range shown in the chart. It can be used for circular and noncircular pipe flows and for open-channel flows



Moody's Chart (Friction factor as a function of Reynolds number and relative roughness for round pipes)



The figure shows the functional dependence of f on Re and k/D and is called the **Moody chart** in honor of **L. F. Moody**, who, along with C. F. Colebrook, correlated the original data of Nikuradse in terms of the relative roughness of commercially available pipe materials

Colebrook's formula

- ▶ A difficulty with its use is that it is implicit in the dependence of f . That is, for given conditions it is not possible to solve for f without some sort of iterative scheme.
- ▶ A word of caution is in order concerning the use of the Moody chart or the equivalent Colebrook formula.
- ▶ Because of various inherent inaccuracies involved (uncertainty in the relative roughness, uncertainty in the experimental data used to produce the Moody chart, etc.), the use of several place accuracy in pipe flow problems is usually not justified.
- ▶ As a rule of thumb, a 10-15% accuracy is the best expected. It is possible to obtain an equation that adequately approximates the Colebrook/Moody chart relationship but does not require an iterative scheme.
- ▶ For example, an alternate form, which is easier to use, is given by

$$\frac{1}{\sqrt{f}} = -1.8 \log \left[\left(\frac{k/D}{3.7} \right)^{1.11} + \frac{6.9}{Re} \right]$$

where one can solve for f explicitly.



Resistance to Flow of Fluids

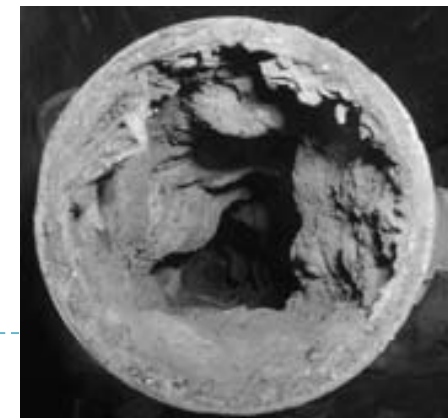
- ▶ In commercially available pipes the roughness is not as uniform and well defined as in the artificially roughened pipes used by Nikuradse. However, it is possible to obtain a measure of the effective relative roughness of typical pipes and thus to obtain the friction factor.

Equivalent Roughness for New Pipes [From Moody and Colebrook]

Pipe	Equivalent Roughness, k	
	Feet	Millimeters
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

From tests with commercial pipes, recommended values for average pipe roughness

-
- ▶ It is important to observe that the values of relative roughness given pertain to new, clean pipes. After considerable use, most pipes (because of a buildup of corrosion or scale) may have a relative roughness that is considerably larger (perhaps by an order of magnitude) than that given.
 - ▶ As shown by the figure in the margin, very old pipes may have enough scale buildup to not only alter the value of but also to change their effective diameter by a considerable amount.



Types of pipe flow problems

- ▶ Given d , L , and V or Q , ρ , μ , and g , compute the head loss h_f (head loss problem).
 - ▶ Calculate Re and k/D from the given data
 - ▶ Obtain f from the Moody's chart
- ▶ Given d , L , h_f , ρ , μ , and g , compute the velocity V or flow rate Q (flow rate problem).
 - ▶ Calculate k/D from the given data and $Re\sqrt{f}$ from \longrightarrow $Re\sqrt{f} = \frac{VD}{\nu} \left(\frac{2gh_f D}{V^2 L} \right)^{1/2}$
 - ▶ Using Coolebrook formula and the above equation, Obtain f
 - ▶ Obtain Re from the Moody's chart and hence Q
- ▶ Given Q , L , h_f , ρ , μ , and g , compute the diameter d of the pipe (sizing problem).
 - ▶ Assume a suitable value of f and calculate Dia from Darcy-Weisbatch equation
 - ▶ With this trial value of D , calculate k/D and Re
 - ▶ With this k/D and Re , calculate f from Moody's diagram
 - ▶ Repeat the process till f becomes same
- ▶ Given Q , d , h_f , ρ , μ , and g , compute the pipe length L .



Problems

- ▶ Compute the loss of head and pressure drop in 200 ft of horizontal 6-in-diameter asphalted cast iron pipe carrying water with a mean velocity of 6 ft/s. (k for asphalted cast iron = 0.0004 ft)
- ▶ Oil, with $\rho = 900 \text{ kg/m}^3$ and $\nu = 0.00001 \text{ m}^2/\text{s}$, flows at $0.2 \text{ m}^3/\text{s}$ through 500 m of 200-mm-diameter cast iron pipe. Determine (a) the head loss and (b) the pressure drop if the pipe slopes down at 10° in the flow direction.
- ▶ Water at 60°F flows through a 6-in.-diameter pipe with an average velocity of 15 ft/s. Approximately what is the height of the largest roughness element allowed if this pipe is to be classified as smooth? [$\nu = 1.21 \times 10^{-5} \text{ ft}^2/\text{s}$]



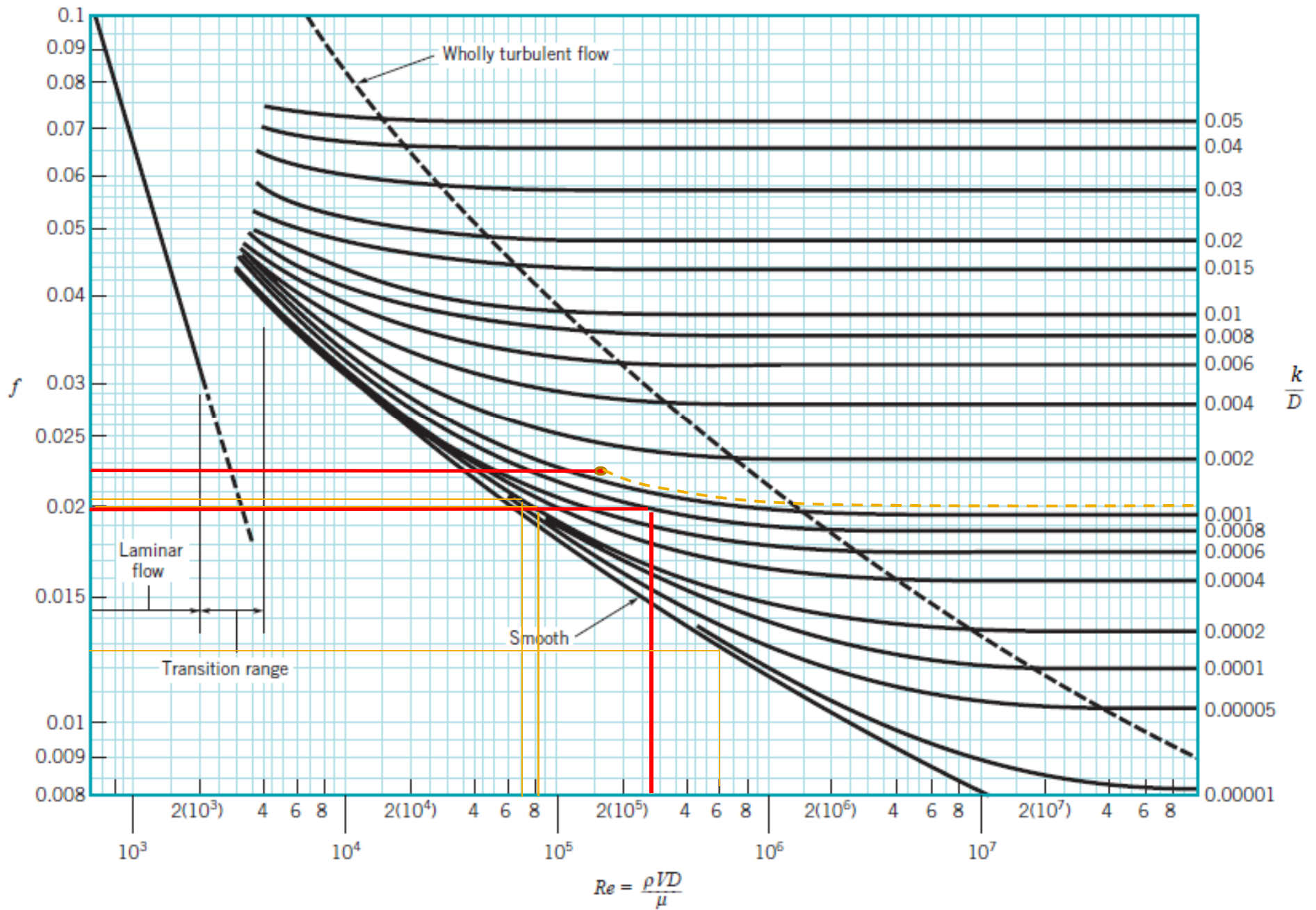
Problem

- ▶ Oil, with $\rho = 950 \text{ kg/m}^3$ and $\nu = 2 \times 10^{-5} \text{ m}^2/\text{s}$, flows through a 30-cm-diameter pipe 100 m long with a head loss of 8 m. The roughness ratio is $k/D = 0.0002$. Find the average velocity and flow rate.
- ▶ Find the wall roughness k if everything else is known: $V = 6 \text{ ft/s}$, $d = 0.5 \text{ ft}$, $L = 200 \text{ ft}$, $\rho = 1.94 \text{ slug/ft}^3$, $\mu = 2.09 \times 10^{-5} \text{ slug/ft-s}$, $h_f = 4.5 \text{ ft}$.
- ▶ A pump delivers 0.6 hp to water at 68°F, flowing in a 6-in-diameter asphalted cast iron horizontal pipe at $V = 6 \text{ ft/s}$. What is the proper pipe length to match these conditions? **[1 hp = 550 ft.lbf/s]**

$$Q = AV = \frac{\pi \times 0.5^2 \times 6}{4} = 1.18 \text{ ft}^3 / \text{s} \quad \longrightarrow \quad h_f = \frac{P}{\rho g Q} = \frac{0.6 \times 550}{1.94 \times 32.3 \times 1.18} = 4.48 \text{ ft}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{1.94 \times 6 \times 0.5}{2.09 \times 10^{-5}} = 2.79 \times 10^5 \quad \frac{k}{D} = \frac{0.0004}{0.5} = 0.0008 \quad \text{From } \frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{k/D}{3.7} + \frac{2.51}{\text{Re} \sqrt{f}} \right) \quad f = 0.0198$$

$$L = \frac{2h_f g D}{f V^2} = 2 \times \frac{4.48 \times 32.2 \times 0.5}{0.0198 \times 6^2} = 202.4 \text{ ft}$$



Problem

- ▶ A 50-mm diameter galvanized iron service pipe is connected to a water main in which the pressure is 450 kPa gage. If the length of the service pipe to a faucet is 40 m and the faucet is 1.2 m above the main, estimate the flowrate when the faucet is fully open.
- ▶ Water at 15 °C is flowing steadily in a 5 cm-diameter horizontal pipe made of stainless steel at a rate of 0.34 m³/min. Determine the pressure drop, the head loss, and the required pumping power input for flow over a 6 m-long section of the pipe.
- ▶ A 15-mm-diameter water pipe is 20 m long and delivers water at 0.0005 m³/sec at 20 °C. What fraction of this pipe is taken up by the entrance region so that after this region fluid flow becomes fully developed? Take $\nu = 1.01 \times 10^{-6}$ m²/sec.



Problem

- ▶ Water from a treatment plant is pumped into a distribution system at a rate of $4.38 \text{ m}^3/\text{s}$, a pressure of 480 kPa, and a temperature of 20°C . The diameter of the pipe is 750 mm and is made of ductile iron. Estimate the pressure 200 m downstream of the treatment plant if the pipeline remains horizontal. After 20 years in operation, scale buildup is expected to cause the equivalent sand roughness of the pipe to increase by a factor of 10. Determine the effect on the water pressure 200 m downstream of the treatment plant.
-



Problem

- ▶ After 15 years of service a steel water main 0.6 m in diameter is found to require 40% more power to deliver the 300 liters/second for which it was originally designed. Determine the corresponding magnitude of the rate of roughness increase α . Take kinematic viscosity of water $\nu = 0.015$ Stokes.
 - ▶ A commercial new galvanized iron service pipe from a water main is required to deliver 200 L/s of water during a fire. If the length of the service pipe is 35 m, the allowable head loss in the pipe is 50 m and kinematic viscosity of water at 20 °C is 1.00×10^{-6} m²/sec, what will the pipe diameter to be used for this purpose?
-



Problem

- ▶ A new reservoir will use gravity to supply drinking water to a water treatment plant serving several surrounding towns as shown in Figure. The required flow rate is $0.315 \text{ m}^3/\text{sec}$. The surface of the reservoir is 61 m above the plain where the water treatment plant is located, and the supply pipe is commercial steel, 914.4 mm in diameter. If the minimum pressure required at the water treatment plant is 347.7 kpa (gage), how far away can the reservoir be located with this size pipe? Assume that minor losses are negligible and that the water is at 283.1 K. The average height of the pipe wall roughness protrusions may be taken as 0.0458 mm. Take kinematic viscosity of water $\nu = 0.13 \times 10^{-5} \text{ m}^2/\text{sec}$.



