Dimensional Analysis, hydraulic similitude and model investigation

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Introduction

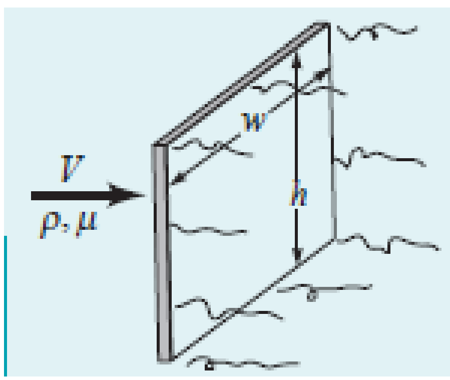
- Although many practical engineering problems involving fluid mechanics can be solved by using the equations and analytical procedures described in the preceding chapters, there remain a large number of problems that rely on experimentally obtained data for their solution.
- In fact, it is probably fair to say that very few problems involving real fluids can be solved by analysis alone. The solution to many problems is achieved through the use of a combination of theoretical and numerical analysis and experimental data.
  - Thus, engineers working on fluid mechanics problems should be familiar with the experimental approach to these problems so that they can interpret and make use of data obtained by others, such as might appear in handbooks, or be able to plan and execute the necessary experiments in their own laboratories.
- In this chapter we consider some techniques and ideas that are important in the planning and execution of experiments, as well as in understanding and correlating data that may have been obtained by other experimenters.

Introduction

- An obvious goal of any experiment is to make the results as widely applicable as possible.
- To achieve this end, the concept of ***similitude*** is often used so that measurements made on one system (for example, in the laboratory) can be used to describe the behavior of other similar systems (outside the laboratory).
- The laboratory systems are usually thought of as *models* and are used to study the phenomenon of interest under carefully controlled conditions. From these model studies, empirical formulations can be developed, or specific predictions of one or more characteristics of some other similar system can be made.
- To do this, it is necessary to establish the relationship between the laboratory model and the "other" system.

Dimensional Analysis

- To illustrate a typical fluid mechanics problem in which experimentation is required, consider A thin rectangular plate having a width *w* and a height *h* is located so that it is normal to a moving stream of fluid.



- An important characteristic of this system, which would be of interest to an engineer is the drag force exerted by the fluid on the plate.

Dimensional Analysis

- The first step in the planning of an experiment to study this problem would be to decide on the factors, or variables, that will have an effect on the drag force (N or lb)
- Let's say, we expect the list to include *w* and *h*, the fluid viscosity and density, μ and ρ respectively, and the velocity *V* of the fluid approaching the plate.
- To perform the experiments in a meaningful and systematic manner, it would be necessary to change one of the variables, such as the velocity, while holding all others constant, and measure the corresponding drag force
- This testing would require $5^4 = 625$ experiments.
- Fortunately, there is a much simpler approach to this problem that will eliminate the difficulties described above i.e. Dimensional Analysis

What do we gain by using Dimensional Analysis?

1. Reduce the number of parameters we need to vary to characterize the problem.
2. Results are independent of the system of units.
3. We don't have to conduct an experiment on every single size of plate at every velocity. Our results will even work for different fluids.
4. Predict trends in the relationship between parameters.

Dimensional Analysis

- Each physical phenomena can be expressed by an equation, composed of variable (or physical quantities) which may be dimensional and non-dimensional quantities. Dimensional Analysis helps in determining a systematic arrangement of variables in the physical relationship and combining dimensional variables to form non-dimensional parameters

Uses:

- Testing the dimensional homogeneity of any equation of fluid motion
- Deriving equations expressed in terms of non-dimensional parameters to show the relative significance of each parameter
  - Given a number of variables and given that they are interrelated, the nature of relation among the variables can be determined by two methods namely,
    - Rayleigh's method, and
    - Buckingham's method

Dimensions and Units

- Review
  - Dimension: Measure of a physical quantity, e.g., length, time, mass
  - Units: Assignment of a number to a dimension, e.g., (m), (sec), (kg)
  - 7 Primary Dimensions:

1.	Mass	m	(kg)
2.	Length	L	(m)
3.	Time	t	(sec)
4.	Temperature	T	(K)
5.	Current	I	(A)
6.	Amount of Light	C	(cd)
7.	Amount of matter	N	(mol)

Dimensions and Units

- Review, continued
  - All non-primary dimensions can be formed by a combination of the 7 primary dimensions
  - Examples
    - {Velocity} = {Length/Time} = {L/t}
    - {Force} = {Mass Length/Time} = $\{mL/t^2\}$

Buckingham's π-method

- The Buckingham's π-theorem states that if there are n dimensional variables involved in a phenomenon, which can be completely described by m fundamental quantities or dimensions (such mass, length, time etc.), and are related by a dimensionally homogeneous equation, then the relationship among the n quantities can always be expressed in terms of exactly (n - m) dimensionless and independent π terms.

Procedure

- List all the physical quantities or variables involved in the phenomenon. Note their dimensions and the number *m* of the fundamental dimensions comprised in them. So that there will be *(n-m)* π-terms
- Select *m* variables out of these which are to serve as repeating variables.
  - These variables should be such that none of them is dimensionless, no two variables have the same dimensions, they themselves do not form a dimensionless parameter and all the *m* fundamental parameters are included collectively in them.
  - Dependent variable should not be taken as a repeating variable
  - In fluid flow problems, usually a characteristic linear dimension, a characteristic velocity and a characteristic fluid property (e.g. density) are chosen as repeating variables.

- Write the general equations for different π-terms.
  - Product of repeating variables each raised to an unknown exponent and one of the remaining variables, taken in turn, with a known power (usually taken as one)
- Write the dimensional equations for the equations of the π-terms obtained in step above.
- Write the final general equation for the phenomenon in terms of the π-terms
  - Any π-term may be replaced by any power of that term, including negative and fractional powers.
  - Any π-term may be replaced by multiplying it by a numerical constant.

Solve

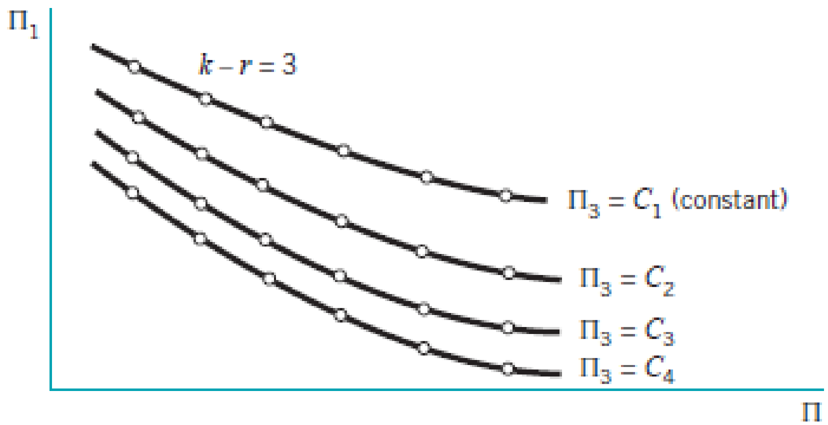
- A thin rectangular plate having a width *w and a height h is located so that it is normal to a moving stream of fluid as* shown in Fig. Assume the drag, d, that the fluid exerts on the plate is a function of *w and h, the fluid viscosity and density, and , respectively, and the velocity V of the fluid approaching* the plate.

Some Common Variables and Dimensionless Groups in Fluid Mechanics

Variables: Acceleration of gravity, g; Bulk modulus, E_v; Characteristic length, ℓ; Density, ρ; Frequency of oscillating flow, ω; Pressure, p (or Δp); Speed of sound, c; Surface tension, σ; Velocity, V; Viscosity, μ

Dimensionless Groups	Name	Interpretation (Index of Force Ratio Indicated)	Types of Applications
$\dfrac{\rho V \ell}{\mu}$	Reynolds number, Re	$\dfrac{\text{inertia force}}{\text{viscous force}}$	Generally of importance in all types of fluid dynamics problems
$\dfrac{V}{\sqrt{g\ell}}$	Froude number, Fr	$\dfrac{\text{inertia force}}{\text{gravitational force}}$	Flow with a free surface
$\dfrac{p}{\rho V^2}$	Euler number, Eu	$\dfrac{\text{pressure force}}{\text{inertia force}}$	Problems in which pressure, or pressure differences, are of interest
$\dfrac{\rho V^2}{E_v}$	Cauchy number,[a] Ca	$\dfrac{\text{inertia force}}{\text{compressibility force}}$	Flows in which the compressibility of the fluid is important
$\dfrac{V}{c}$	Mach number,[a] Ma	$\dfrac{\text{inertia force}}{\text{compressibility force}}$	Flows in which the compressibility of the fluid is important
$\dfrac{\omega \ell}{V}$	Strouhal number, St	$\dfrac{\text{inertia (local) force}}{\text{inertia (convective) force}}$	Unsteady flow with a characteristic frequency of oscillation
$\dfrac{\rho V^2 \ell}{\sigma}$	Weber number, We	$\dfrac{\text{inertia force}}{\text{surface tension force}}$	Problems in which surface tension is important

Correlation of experimental data



The graphical presentation of data for problems involving three pi terms.

- As the number of pi terms continues to increase, corresponding to an increase in the general complexity of the problem of interest, both the graphical presentation and the determination of a suitable empirical equation become intractable.
- For these more complicated problems, it is often more feasible to use models to predict specific characteristics of the system rather than to try to develop general correlations.

Modeling and Similitude

- Major engineering projects involving structures, aircraft, ships, rivers, harbors, dams, air and water pollution, and so on, frequently involve the use of models.
- *A **model*** is a representation of a physical system that may be used to predict the behavior of the system in some desired respect. The physical system for which the predictions are to be made is called the ***prototype*.**
- Although mathematical or computer models may also conform to this definition, our interest will be in physical models, that is, models that resemble the prototype but are generally of a different size, may involve different fluids, and often operate under different conditions (pressures, velocities, etc.).

Modeling and Similitude

- Usually a model is smaller than the prototype. Therefore, it is more easily handled in the laboratory and less expensive to construct and operate than a large prototype
- Occasionally, if the prototype is very small, it may be advantageous to have a model that is larger than the prototype so that it can be more easily studied.
  - For example, large models have been used to study the motion of red blood cells, which are approximately 8 μm in diameter.

A 1 : 46.6 scale model of an U.S. Navy fleet destroyer being tested in the 100-m long towing tank at the University of Iowa. The model is 3.048 m long.

In tests like this, the Froude number is the most important Non-dimensional parameter.

There is more to the design than simply scaling the geometry!

- The theory of models can be readily developed by using the principles of dimensional analysis.

$$\Pi_1 = \phi(\Pi_2, \Pi_3, \ldots, \Pi_n)$$

- If the above equation describes the behavior of a particular prototype, a similar relationship can be written for a model of this prototype

$$\Pi_{1m} = \phi(\Pi_{2m}, \Pi_{3m}, \ldots, \Pi_{nm})$$

- The π-terms can be developed so that π_1 contains the variable that is to be predicted from observations made on the model.

There is more to the design than simply scaling the geometry!

$$\boxed{\begin{aligned} \Pi_{2m} &= \Pi_2 \\ \Pi_{3m} &= \Pi_3 \\ &\vdots \\ \Pi_{nm} &= \Pi_n \end{aligned}}$$

- With the presumption that the form of ϕ is the same for model and prototype, it follows that

$$\boxed{\Pi_1 = \Pi_{1m}}$$

The Principle of Similarity

Three necessary conditions for complete similarity between a model and a prototype are:

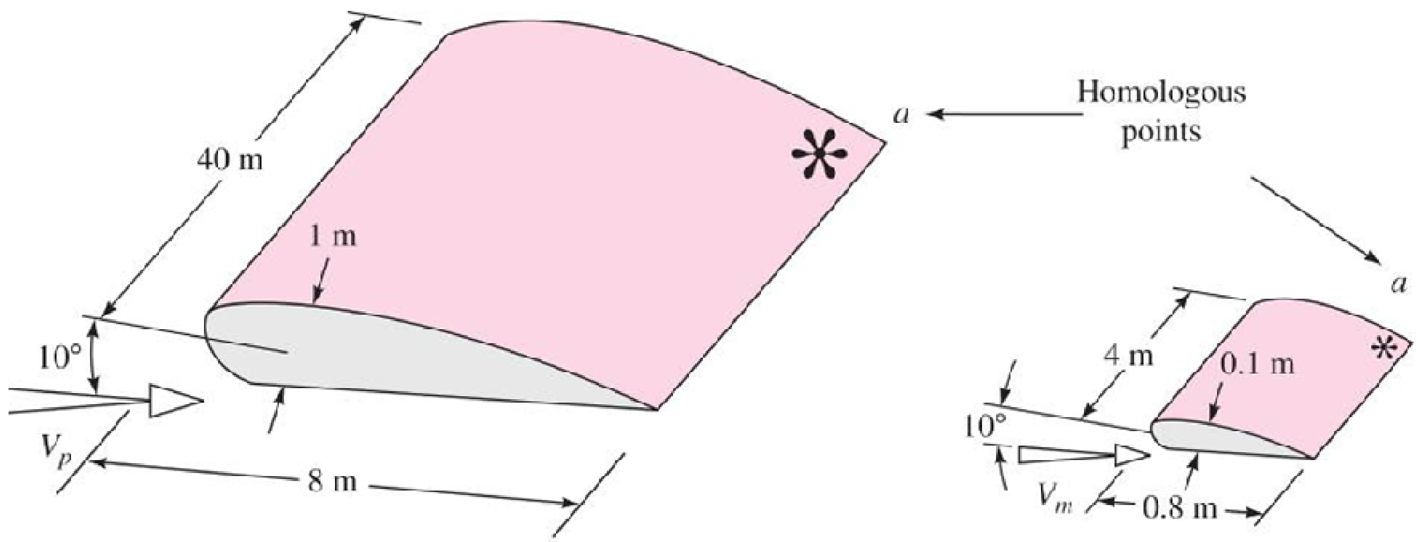
1. Geometric similarity –

- Ratio of significant dimensions should be same in two systems (i.e. Prototype and Model).
- Similarity of shape.

The Principle of Similarity

Geometric similarity –

A model and prototype are geometric similar if and only if all body dimension in all three coordinates have the same linear scale ratio.



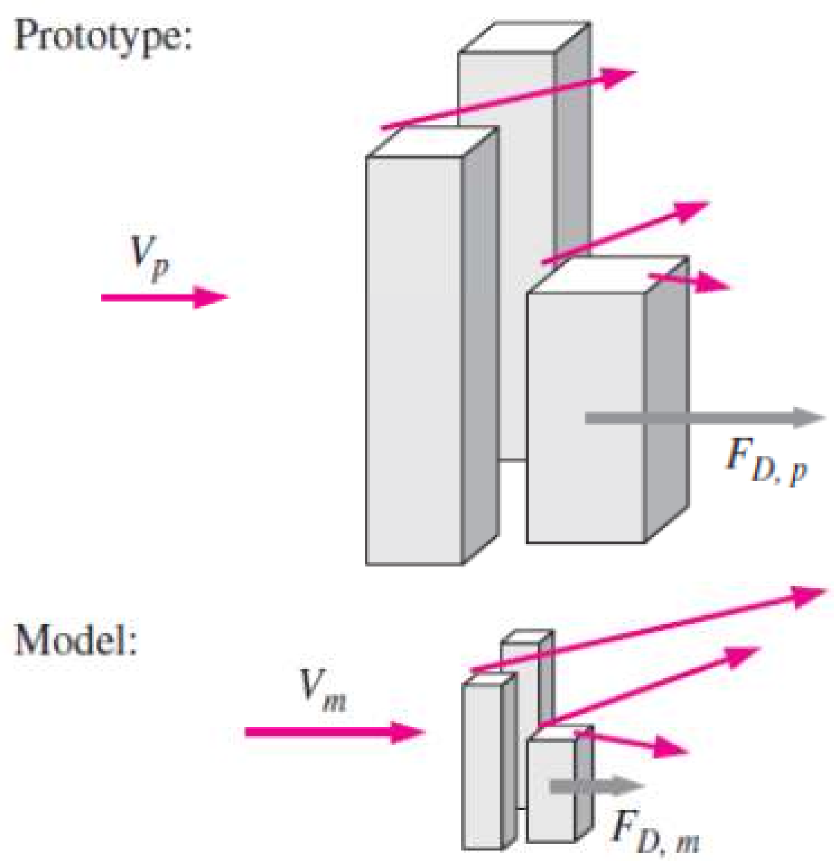
(a) (b)

The Principle of Similarity

2. Kinematic similarity –

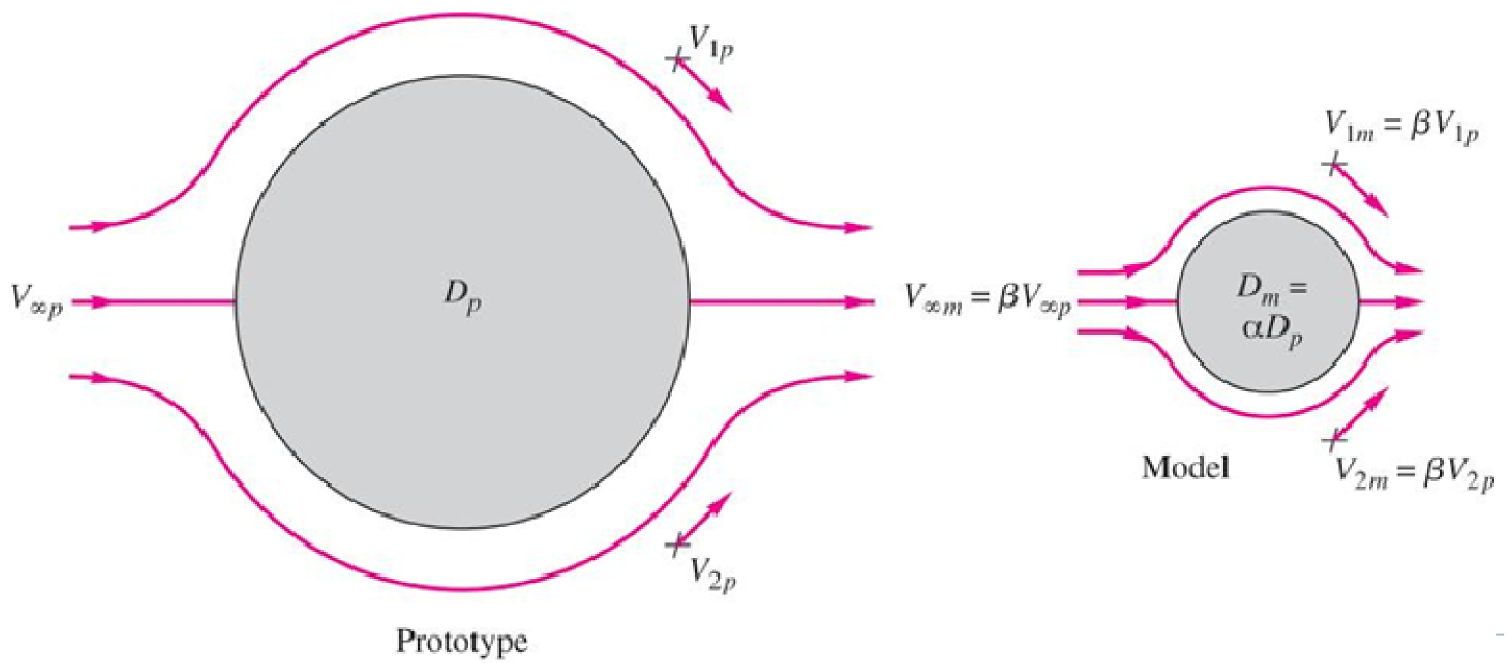
the velocity at any point in the model flow must be proportional (by a constant scale factor) to the velocity at the corresponding point in the prototype flow.

- Similarity of motion.



The Principle of Similarity

The motions of two systems are Kinematically similar if homologous particles lie at homologous points at homologous time.



(a)

The Principle of Similarity

3. Dynamic similarity –

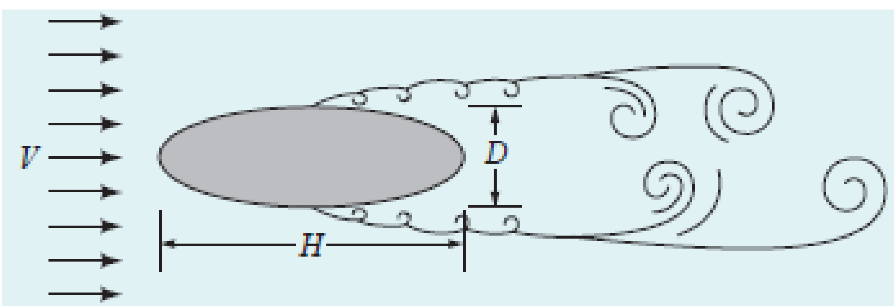
- When all forces in the model flow scale by a constant factor to corresponding forces in the prototype flow (force-scale equivalence).
- Similarity of Forces

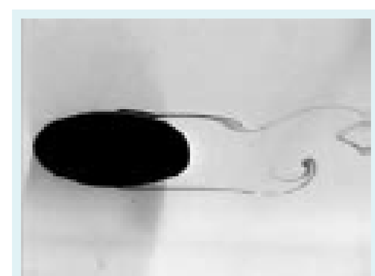
Solve..

- A long structural component of a bridge has an elliptical cross section shown in Fig. It is known that when a steady wind blows past this type of bluff body, vortices may develop on the downwind side that are shed in a regular fashion at some definite frequency. Since these vortices can create harmful periodic forces acting on the structure, it is important to determine the shedding frequency. For the specific structure of interest, D=0.1 m, H=0.3 m, and a representative wind velocity is 50 km/hr. Standard air can be assumed. The shedding frequency is to be determined through the use of a small-scale model that is to be tested in a water tunnel. For the model D_m = 20 mm and the water temperature is 20°C.

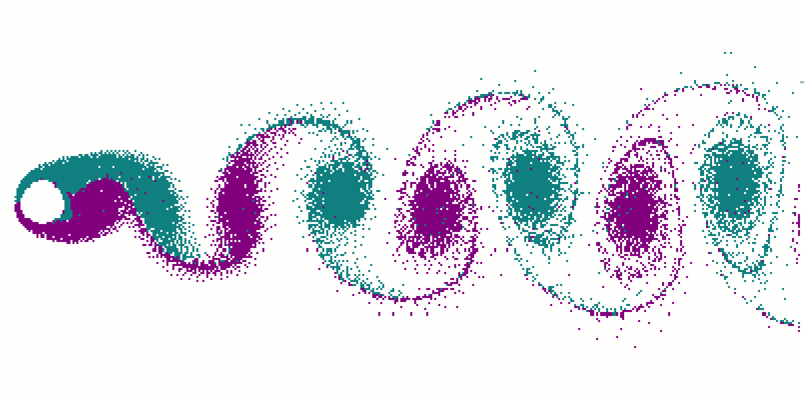
  Determine the model dimension, and the velocity at which the test should be performed. If the shedding frequency for the model is found to be 49.9 Hz, what is the corresponding frequency for the prototype?

[$\mu_{air} = 1.79 \times 10^{-5}$ kg/ms; $\rho = 1.23$ kg/m^3; $\mu_{water\ at\ 20°C} = 10^{-3}$ kg/ms; $\rho = 998$ kg/m^3]





- The shedding frequency is expected to depend on the lengths D and H, the approach velocity, V, and the fluid density, ρ and viscosity μ.



Vortex shedding is an unsteady oscillating flow that takes place when a fluid such as air or water flows past a blunt cylindrical body at certain velocities, depending to the size and shape of the body. In this flow, vortices are created at the back of the body and detach periodically from either side of the body.

If the cylindrical structure is not mounted rigidly and the frequency of vortex shedding matches the resonance frequency of the structure, the structure can begin to resonate, vibrating with harmonic oscillations driven by the energy of the flow.

- **Strouhal number** is a dimensionless number describing oscillating flow mechanisms

$$\mathrm{St} = \frac{fL}{V},$$

where S_t is the dimensionless Strouhal number, f is the frequency of vortex shedding, L is the characteristic length (for example hydraulic diameter) and V is the velocity of the fluid.

Practice Problems

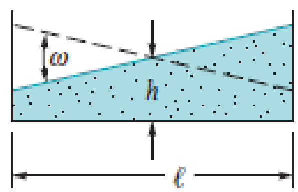
- The pressure rise, Δp, across a pump can be expressed as

$$\Delta p = f\left(D, \rho, \omega, Q\right)$$

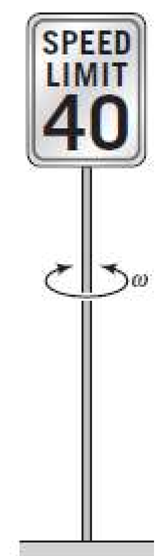
  - where D is the impeller diameter, ρ the fluid density, ω the rotational speed, and Q the flowrate. Determine a suitable set of dimensionless parameters.
- A thin elastic wire is placed between rigid supports. A fluid flows past the wire, and it is desired to study the static deflection, δ, at the center of the wire due to the fluid drag. Assume that $\delta = f\left(l, d, \rho, \mu, V, E\right)$
  - where l is the wire length, d the wire diameter, ρ the fluid density, μ the fluid viscosity, V the fluid velocity, and E the modulus of elasticity of the wire material. Develop a suitable set of pi terms for this problem.

Practice Problems

- Water sloshes back and forth in a tank as shown in Fig. The frequency of sloshing, ω, is assumed to be a function of the acceleration of gravity, g, the average depth of the water, h, and the length of the tank, l. Develop a suitable set of dimensionless parameters for this problem using g and l as repeating variables.



- Under certain conditions, wind blowing past a rectangular speed limit sign can cause the sign to oscillate with a frequency ω (See Fig.) Assume that ω is a function of the sign width, *b,* sign height, *h,* wind velocity, *V,* air density, ρ and an elastic constant, *k,* for the supporting pole. The constant, *k*, has dimensions of FL. Develop a suitable set of pi terms for this problem.



Practice Problems

- A model of a submarine, 1 : 15 scale, is to be tested at 180 ft/s in a wind tunnel with standard sea-level air, while the prototype will be operated in seawater. Determine the speed of the prototype to ensure Reynolds number similarity.
- The drag characteristics of a torpedo are to be studied in a water tunnel using a 1:5 scale model. The tunnel operates with freshwater at 20°C, whereas the prototype torpedo is to be used in seawater at 15.6°C. To correctly simulate the behavior of the prototype moving with a velocity of 30 m/s, what velocity is required in the water tunnel?
- The fluid dynamic characteristics of an airplane flying 240 mph at 10,000 ft are to be investigated with the aid of a 1 : 20 scale model. If the model tests are to be performed in a wind tunnel using standard air, what is the required air velocity in the wind tunnel? Is this a realistic velocity?

- A prototype automobile is designed to travel at 65 km hr. A model of this design is tested in a wind tunnel with identical standard sea-level air properties at a 1 : 5 scale. The measured model drag is 400 N, enforcing dynamic similarity. Determine (a) the drag force on the prototype and (b) the power required to overcome this drag.

Physical Properties of Water (BG Units)[a]

Temperature (°F)	Density, ρ (slugs/ft^3)	Specific Weight[b], γ (lb/ft^3)	Dynamic Viscosity, μ (lb·s/ft^2)	Kinematic Viscosity, ν (ft^2/s)	Surface Tension[c], σ (lb/ft)	Vapor Pressure, p_v [lb/in.^{2}(abs)]	Speed of Sound[d], c (ft/s)
32	1.940	62.42	3.732 E − 5	1.924 E − 5	5.18 E − 3	8.854 E − 2	4603
40	1.940	62.43	3.228 E − 5	1.664 E − 5	5.13 E − 3	1.217 E − 1	4672
50	1.940	62.41	2.730 E − 5	1.407 E − 5	5.09 E − 3	1.781 E − 1	4748
60	1.938	62.37	2.344 E − 5	1.210 E − 5	5.03 E − 3	2.563 E − 1	4814
70	1.936	62.30	2.037 E − 5	1.052 E − 5	4.97 E − 3	3.631 E − 1	4871
80	1.934	62.22	1.791 E − 5	9.262 E − 6	4.91 E − 3	5.069 E − 1	4819
90	1.931	62.11	1.500 E − 5	8.233 E − 6	4.86 E − 3	6.979 E − 1	4960
100	1.927	62.00	1.423 E − 5	7.383 E − 6	4.79 E − 3	9.493 E − 1	4995
120	1.918	61.71	1.164 E − 5	6.067 E − 6	4.67 E − 3	1.692 E + 0	5049
140	1.908	61.38	9.743 E − 6	5.106 E − 6	4.53 E − 3	2.888 E + 0	5091
160	1.896	61.00	8.315 E − 6	4.385 E − 6	4.40 E − 3	4.736 E + 0	5101
180	1.883	60.58	7.207 E − 6	3.827 E − 6	4.26 E − 3	7.507 E + 0	5195
200	1.869	60.12	6.342 E − 6	3.393 E − 6	4.12 E − 3	1.152 E + 1	5089
212	1.860	59.83	5.886 E − 6	3.165 E − 6	4.04 E − 3	1.469 E + 1	5062

Physical Properties of Water (SI Units)[a]

Temperature (°C)	Density, ρ (kg/m^3)	Specific Weight[b], γ (kN/m^3)	Dynamic Viscosity, μ (N·s/m^2)	Kinematic Viscosity, ν (m^2/s)	Surface Tension[c], σ (N/m)	Vapor Pressure, p_v [N/m^2(abs)]	Speed of Sound[d], c (m/s)
0	999.9	9.806	1.787 E − 3	1.787 E − 6	7.56 E − 2	6.105 E + 2	1403
5	1000.0	9.807	1.519 E − 3	1.519 E − 6	7.49 E − 2	8.722 E + 2	1427
10	999.7	9.804	1.307 E − 3	1.307 E − 6	7.42 E − 2	1.228 E + 3	1447
20	998.2	9.789	1.002 E − 3	1.004 E − 6	7.28 E − 2	2.338 E + 3	1481
30	995.7	9.765	7.975 E − 4	8.009 E − 7	7.12 E − 2	4.243 E + 3	1507
40	992.2	9.731	6.529 E − 4	6.580 E − 7	6.96 E − 2	7.376 E + 3	1526
50	988.1	9.690	5.468 E − 4	5.534 E − 7	6.79 E − 2	1.233 E + 4	1541
60	983.2	9.642	4.665 E − 4	4.745 E − 7	6.62 E − 2	1.992 E + 4	1552
70	977.8	9.589	4.042 E − 4	4.134 E − 7	6.44 E − 2	3.116 E + 4	1555
80	971.8	9.530	3.547 E − 4	3.650 E − 7	6.26 E − 2	4.734 E + 4	1555
90	965.3	9.467	3.147 E − 4	3.260 E − 7	6.08 E − 2	7.010 E + 4	1550
100	958.4	9.399	2.818 E − 4	2.940 E − 7	5.89 E − 2	1.013 E + 5	1543

Physical Properties of Air at Standard Atmospheric Pressure (BG Units)[a]

Temperature (°F)	Density, ρ (slugs/ft^3)	Specific Weight[b], γ (lb/ft^3)	Dynamic Viscosity, μ (lb·s/ft^2)	Kinematic Viscosity, ν (ft^2/s)	Specific Heat Ratio, k (—)	Speed of Sound, c (ft/s)
−40	2.939 E − 3	9.456 E − 2	3.29 E − 7	1.12 E − 4	1.401	1004
−20	2.805 E − 3	9.026 E − 2	3.34 E − 7	1.19 E − 4	1.401	1028
0	2.683 E − 3	8.633 E − 2	3.38 E − 7	1.26 E − 4	1.401	1051
10	2.626 E − 3	8.449 E − 2	3.44 E − 7	1.31 E − 4	1.401	1062
20	2.571 E − 3	8.273 E − 2	3.50 E − 7	1.36 E − 4	1.401	1074
30	2.519 E − 3	8.104 E − 2	3.58 E − 7	1.42 E − 4	1.401	1085
40	2.469 E − 3	7.942 E − 2	3.60 E − 7	1.46 E − 4	1.401	1096
50	2.420 E − 3	7.786 E − 2	3.68 E − 7	1.52 E − 4	1.401	1106
60	2.373 E − 3	7.636 E − 2	3.75 E − 7	1.58 E − 4	1.401	1117
70	2.329 E − 3	7.492 E − 2	3.82 E − 7	1.64 E − 4	1.401	1128
80	2.286 E − 3	7.353 E − 2	3.86 E − 7	1.69 E − 4	1.400	1138
90	2.244 E − 3	7.219 E − 2	3.90 E − 7	1.74 E − 4	1.400	1149
100	2.204 E − 3	7.090 E − 2	3.94 E − 7	1.79 E − 4	1.400	1159
120	2.128 E − 3	6.846 E − 2	4.02 E − 7	1.89 E − 4	1.400	1180
140	2.057 E − 3	6.617 E − 2	4.13 E − 7	2.01 E − 4	1.399	1200
160	1.990 E − 3	6.404 E − 2	4.22 E − 7	2.12 E − 4	1.399	1220
180	1.928 E − 3	6.204 E − 2	4.34 E − 7	2.25 E − 4	1.399	1239
200	1.870 E − 3	6.016 E − 2	4.49 E − 7	2.40 E − 4	1.398	1258
300	1.624 E − 3	5.224 E − 2	4.97 E − 7	3.06 E − 4	1.394	1348
400	1.435 E − 3	4.616 E − 2	5.24 E − 7	3.65 E − 4	1.389	1431
500	1.285 E − 3	4.135 E − 2	5.80 E − 7	4.51 E − 4	1.383	1509
750	1.020 E − 3	3.280 E − 2	6.81 E − 7	6.68 E − 4	1.367	1685
1000	8.445 E − 4	2.717 E − 2	7.85 E − 7	9.30 E − 4	1.351	1839
1500	6.291 E − 4	2.024 E − 2	9.50 E − 7	1.51 E − 3	1.329	2114

Physical Properties of Air at Standard Atmospheric Pressure (SI Units)[a]

Temperature (°C)	Density, ρ (kg/m^3)	Specific Weight[b], γ (N/m^3)	Dynamic Viscosity, μ (N·s/m^2)	Kinematic Viscosity, ν (m^2/s)	Specific Heat Ratio, k (—)	Speed of Sound, c (m/s)
−40	1.514	14.85	1.57 E − 5	1.04 E − 5	1.401	306.2
−20	1.395	13.68	1.63 E − 5	1.17 E − 5	1.401	319.1
0	1.292	12.67	1.71 E − 5	1.32 E − 5	1.401	331.4
5	1.269	12.45	1.73 E − 5	1.36 E − 5	1.401	334.4
10	1.247	12.23	1.76 E − 5	1.41 E − 5	1.401	337.4
15	1.225	12.01	1.80 E − 5	1.47 E − 5	1.401	340.4
20	1.204	11.81	1.82 E − 5	1.51 E − 5	1.401	343.3
25	1.184	11.61	1.85 E − 5	1.56 E − 5	1.401	346.3
30	1.165	11.43	1.86 E − 5	1.60 E − 5	1.400	349.1
40	1.127	11.05	1.87 E − 5	1.66 E − 5	1.400	354.7
50	1.109	10.88	1.95 E − 5	1.76 E − 5	1.400	360.3
60	1.060	10.40	1.97 E − 5	1.86 E − 5	1.399	365.7
70	1.029	10.09	2.03 E − 5	1.97 E − 5	1.399	371.2
80	0.9996	9.803	2.07 E − 5	2.07 E − 5	1.399	376.6
90	0.9721	9.533	2.14 E − 5	2.20 E − 5	1.398	381.7
100	0.9461	9.278	2.17 E − 5	2.29 E − 5	1.397	386.9
200	0.7461	7.317	2.53 E − 5	3.39 E − 5	1.390	434.5
300	0.6159	6.040	2.98 E − 5	4.84 E − 5	1.379	476.3
400	0.5243	5.142	3.32 E − 5	6.34 E − 5	1.368	514.1
500	0.4565	4.477	3.64 E − 5	7.97 E − 5	1.357	548.8
1000	0.2772	2.719	5.04 E − 5	1.82 E − 4	1.321	694.8

Approximate Physical Properties of Some Common Liquids (SI Units)

Liquid	Temperature (°C)	Density, ρ (kg/m^3)	Specific Weight, γ (kN/m^3)	Dynamic Viscosity, μ (N · s/m^2)	Kinematic Viscosity, ν (m^2/s)	Surface Tension,[a] σ (N/m)	Vapor Pressure, p_v [N/m^2 (abs)]	Bulk Modulus,[b] E_v (N/m^2)
Carbon tetrachloride	20	1,590	15.6	9.58 E − 4	6.03 E − 7	2.69 E − 2	1.3 E + 4	1.31 E + 9
Ethyl alcohol	20	789	7.74	1.19 E − 3	1.51 E − 6	2.28 E − 2	5.9 E + 3	1.06 E + 9
Gasoline[c]	15.6	680	6.67	3.1 E − 4	4.6 E − 7	2.2 E − 2	5.5 E + 4	1.3 E + 9
Glycerin	20	1,260	12.4	1.50 E + 0	1.19 E − 3	6.33 E − 2	1.4 E − 2	4.52 E + 9
Mercury	20	13,600	133	1.57 E − 3	1.15 E − 7	4.66 E − 1	1.6 E − 1	2.85 E + 10
SAE 30 oil[c]	15.6	912	8.95	3.8 E − 1	4.2 E − 4	3.6 E − 2	—	1.5 E + 9
Seawater	15.6	1,030	10.1	1.20 E − 3	1.17 E − 6	7.34 E − 2	1.77 E + 3	2.34 E + 9
Water	15.6	999	9.80	1.12 E − 3	1.12 E − 6	7.34 E − 2	1.77 E + 3	2.15 E + 9

Approximate Physical Properties of Some Common Gases at Standard Atmospheric Pressure (BG Units)

Gas	Temperature (°F)	Density, ρ (slugs/ft^3)	Specific Weight, γ (lb/ft^3)	Dynamic Viscosity, μ (lb · s/ft^2)	Kinematic Viscosity, ν (ft^2/s)	Gas Constant,[a] R (ft · lb/slug · °R)	Specific Heat Ratio,[b] k
Air (standard)	59	2.38 E − 3	7.65 E − 2	3.74 E − 7	1.57 E − 4	1.716 E + 3	1.40
Carbon dioxide	68	3.55 E − 3	1.14 E − 1	3.07 E − 7	8.65 E − 5	1.130 E + 3	1.30
Helium	68	3.23 E − 4	1.04 E − 2	4.09 E − 7	1.27 E − 3	1.242 E + 4	1.66
Hydrogen	68	1.63 E − 4	5.25 E − 3	1.85 E − 7	1.13 E − 3	2.466 E + 4	1.41
Methane (natural gas)	68	1.29 E − 3	4.15 E − 2	2.29 E − 7	1.78 E − 4	3.099 E + 3	1.31
Nitrogen	68	2.26 E − 3	7.28 E − 2	3.68 E − 7	1.63 E − 4	1.775 E + 3	1.40
Oxygen	68	2.58 E − 3	8.31 E − 2	4.25 E − 7	1.65 E − 4	1.554 E + 3	1.40

Approximate Physical Properties of Some Common Gases at Standard Atmospheric Pressure (SI Units)

Gas	Temperature (°C)	Density, ρ (kg/m^3)	Specific Weight, γ (N/m^3)	Dynamic Viscosity, μ ($N \cdot s/m^2$)	Kinematic Viscosity, ν (m^2/s)	Gas Constant,[a] R ($J/kg \cdot K$)	Specific Heat Ratio,[b] k
Air (standard)	15	1.23 E + 0	1.20 E + 1	1.79 E − 5	1.46 E − 5	2.869 E + 2	1.40
Carbon dioxide	20	1.83 E + 0	1.80 E + 1	1.47 E − 5	8.03 E − 6	1.889 E + 2	1.30
Helium	20	1.66 E − 1	1.63 E + 0	1.94 E − 5	1.15 E − 4	2.077 E + 3	1.66
Hydrogen	20	8.38 E − 2	8.22 E − 1	8.84 E − 6	1.05 E − 4	4.124 E + 3	1.41
Methane (natural gas)	20	6.67 E − 1	6.54 E + 0	1.10 E − 5	1.65 E − 5	5.183 E + 2	1.31
Nitrogen	20	1.16 E + 0	1.14 E + 1	1.76 E − 5	1.52 E − 5	2.968 E + 2	1.40
Oxygen	20	1.33 E + 0	1.30 E + 1	2.04 E − 5	1.53 E − 5	2.598 E + 2	1.40

Properties of the U.S. Standard Atmosphere (BG Units)[a]

Altitude (ft)	Temperature (°F)	Acceleration of Gravity, g (ft/s^2)	Pressure, p [lb/in.^{2}(abs)]	Density, ρ (slugs/ft^3)	Dynamic Viscosity, μ (lb·s/ft^2)
−5,000	76.84	32.189	17.554	2.745 E − 3	3.836 E − 7
0	59.00	32.174	14.696	2.377 E − 3	3.737 E − 7
5,000	41.17	32.159	12.228	2.048 E − 3	3.637 E − 7
10,000	23.36	32.143	10.108	1.756 E − 3	3.534 E − 7
15,000	5.55	32.128	8.297	1.496 E − 3	3.430 E − 7
20,000	−12.26	32.112	6.759	1.267 E − 3	3.324 E − 7
25,000	−30.05	32.097	5.461	1.066 E − 3	3.217 E − 7
30,000	−47.83	32.082	4.373	8.907 E − 4	3.107 E − 7
35,000	−65.61	32.066	3.468	7.382 E − 4	2.995 E − 7
40,000	−69.70	32.051	2.730	5.873 E − 4	2.969 E − 7
45,000	−69.70	32.036	2.149	4.623 E − 4	2.969 E − 7
50,000	−69.70	32.020	1.692	3.639 E − 4	2.969 E − 7
60,000	−69.70	31.990	1.049	2.256 E − 4	2.969 E − 7
70,000	−67.42	31.959	0.651	1.392 E − 4	2.984 E − 7
80,000	−61.98	31.929	0.406	8.571 E − 5	3.018 E − 7
90,000	−56.54	31.897	0.255	5.610 E − 5	3.052 E − 7
100,000	−51.10	31.868	0.162	3.318 E − 5	3.087 E − 7
150,000	19.40	31.717	0.020	3.658 E − 6	3.511 E − 7
200,000	−19.78	31.566	0.003	5.328 E − 7	3.279 E − 7
250,000	−88.77	31.415	0.000	6.458 E − 8	2.846 E − 7

[a]Data abridged from *U.S. Standard Atmosphere*, 1976, U.S. Government Printing Office, Washington, D.C.